Betti numbers of semialgebraic and sub-Pfaffian sets.

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Let X be a subset in $[-1,1]^{n_0} \subset \mathbb{R}^{n_0}$ defined by a formula

$$X = \{\mathbf{x}_0 |\, Q_1 \mathbf{x}_1 Q_2 \mathbf{x}_2 \cdots Q_q \mathbf{x}_q ((\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_q) \in X_q)\},$$

where $Q_i \in \{\exists, \forall\}$, $Q_i \neq Q_{i+1}$, $\mathbf{x}_i \in \mathbb{R}^{n_i}$, and X_q be either an open or a closed set in $[-1,1]^{n_0+\cdots+n_q}$ having a homotopy type of a CW-complex. We express an upper bound on each Betti number of X via a sum of Betti numbers of some sets defined by quantifier-free formulae involving X_q.

In important particular cases of semialgebraic and semi-Pfaffian sets defined by quantifier-free formulae with polynomials and Pfaffian functions respectively, upper bounds on Betti numbers of X_q are well known. Our results allow to extend the bounds to semialgebraic sets defined by formulas with quantifiers, and to sub-Pfaffian sets.