ON ARITHMETIC ASPECTS OF SPARSE RESULTANTS

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Let $\mathcal{A}_0, \ldots, \mathcal{A}_n \subset \mathbb{Z}^n$ be finite sets of integer vectors, and $\mathrm{Res}_{\mathcal{A}_0,\ldots,\mathcal{A}_n} \in \mathbb{Z}[U_0, \ldots, U_n]$ the associated mixed sparse resultant, which is a polynomial in $n+1$ groups $U_i := \{U_{ia}; a \in \mathcal{A}_i\}$ of $m_i := \#\mathcal{A}_i$ variables each.

Resultants are of fundamental importance for solving systems of polynomial equations and therefore have been extensively studied. Recent research has focused on arithmetic aspects of this polynomial such as its *height* and its *Mahler measure*

The absolute height of a polynomial $g = \sum_\alpha c_\alpha U^\alpha \in \mathbb{C}[U]$ is defined as $H(g) := \max\{|c_\alpha|, \alpha \in \mathbb{N}^m\}$, where $U = \{U_{ia}, i = 0, \ldots, n, a \in \mathcal{A}_i\}$ and $m := m_0 + \ldots + m_n$. Its (logarithmic) height is

$$h(g) := \log H(g) = \log \max\{|c_\alpha|, \alpha \in \mathbb{N}^m\}.$$

The Mahler measure of g is defined as

$$m(g) := \frac{1}{(2\pi \mathbb{I})^{n+1}} \int_{\mathbb{T}^{n+1}} \log |g(U_0, \ldots, U_n)| \frac{\mathrm{d}U_0}{U_0} \cdots \frac{\mathrm{d}U_n}{U_n},$$

where for $j = 0, \ldots, n$, $\frac{\mathrm{d}U_j}{U_j}$ is short for $\prod_{a \in \mathcal{A}_j} \frac{\mathrm{d}U_{ja}}{U_{ja}}$, and

$$\mathbb{T}^{n+1} = \{(z_0, \ldots, z_n) \in \mathbb{C}^{n+1} | \, |z_0| = \cdots = |z_n| = 1\}$$

is the $n+1$-torus.

Even though there are explicit upper bounds for both the height and the Mahler measure of resultants, very little seems to be known about the problem of computing explicitly both the height and the Mahler measure of the resultant.

In this talk, we will show how these two objects are related, and also some non trivial heights (joint work with Kevin Hare) and Mahler measures (joint work with Matilde Lalin) of resultants.