

The non-homogeneous p -Laplace equation: equivalence of viscosity and weak solutions

MARÍA MEDINA DE LA TORRE *

Abstract

We study the relation between viscosity and weak solutions for non-homogeneous p -Laplace equations with a lower order term depending on x , u and ∇u , both in the degenerate and in the singular case, that is, we consider the equations

$$-\operatorname{div}(|\nabla u|^{p-2}\nabla u) = f(x, u, \nabla u),$$

defined in an open and bounded set $\Omega \subset \mathbb{R}^n$ and for $1 < p < \infty$.

We prove that any locally bounded viscosity solution constitutes a weak solution, extending results presented in Juutinen, Lindqvist and Manfredi [2] and Julin and Juutinen [1] for the homogeneous case. Moreover, we provide a converse statement in the full case under extra assumptions on the data.

This work can be found in [3].

References

- [1] V. Julin, P. Juutinen, *A new proof for the equivalence of weak and viscosity solutions for the p -Laplace equation*. Communications in PDE **37** 5 (2012), 934-946.
- [2] P. Juutinen, P. Lindqvist, and J. Manfredi, *On the equivalence of viscosity solutions and weak solutions for a quasilinear equation*, SIAM J. Math. Anal. *33* 3 (2001), 699-717.
- [3] M. Medina, P. Ochoa, *On viscosity and weak solutions for non-homogeneous p -Laplace equations*, Advances in Nonlinear Analysis (2017), doi: 10.1515/anona-2017-0005.

*Departamento de Matemática, Pontificia Universidad Católica de Chile, e-mail: mamedinad@mat.uc.cl