# IV International Symposium on Nonlinear PDEs & Free Boundary Problems

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# Posters

**Theory of affine shells: invariant estimates**

Salvador Gigena, Daniel Abud and Moisés Binia

An affine shell, as we have defined it in previous articles [1, 2, 3], made of a perfectly elastic, homogeneous and isotropic material, is subjected to forces acting along its edge, thus passing from an original unstrained state to a final "strained" one. Our objective in this work is to approximate the geometric objects in order to asses how much the shell has been deformed. For achieving that goal we use Non Linear P.D.E. Methods. In particular, equations of MongeAmpere type are considered. Besides, we are interested in knowing how it works the relationship with classical shells, by comparing the same case with that treated previously within the euclidean geometry context, already considered by other authors [4, 5, 6]. In order to perform the job we will use, systematically, the well known unimodular affine geometric invariants, as a frame of reference for these estimates.

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**Looking for the numerical solution of a variational problem related to infinity laplacian**

L. Aragone, P.A. Lotito and L.A. Parente

It is prooved in [2] that u is a solution in the viscosity sense of the infinity laplacian if and only if it is an absolut minimizer of the variational problem consisting in looking for u that minimize the sup ess of the square of the gradient's norm. Taking in consideration this characterization, we present a numerical approximation using finite differences.
We applied this results to the reconstruction of a riverbed. This problem consists in finding an interpolation surface of the known values. When the interpolation criterium is to minimize the maximum of the gradient's norm the mentioned method can be applied.

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**Second order conditions in optimal singular control problems. Applications**

María Soledad Aronna, J. Frederic Bonnans and Pablo Lotito.

We consider an optimal control problem of optimal hydrothermal scheduling. We follow the model discussed in [1]. Our model is deterministic, continuous in time and includes an efficiency function of the turbines with respect to the water volume in valleys. We apply the tools of optimal control theory, particularly, we focus on the analysis of singular arcs. Our main result is a characterization of the Goh-Legendre (GL) condition [2, 3]. As a consequence we show that for some choices of the efficiency coefficients this condition always holds (resp. does not hold). When the GL condition holds, the algebraic variables (controls) can be eliminated from some algebraic expressions and expressed as functions of the differential variables (state and costate). Thus, we are able to give conditions under which these conditions either automatically satisfied or excluded. We present numerical examples that support those results.

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**Maximizing survival probability of an Insurance Company: A free-boundary problem**

Pablo Azcue and Nora Muler

We model the uncontrolled reserve of an insurance company as a compound Poisson process, that is $X_t = x + pt - \sum_{i=1}^{N_t} U_i$ with $x$ the initial reserve, $p$ the premium rate, $U_i$ the size of the claims ( i.i.d. with distribution $F$ ) and $N_t$ a Poisson process with intensity $\beta$. We assume that the management has the possibility of investing part of its reserve in a non-liquid asset while the rest is kept in cash.
We assume that the yield of the non-liquid asset $r$ is positive but there is a transaction cost $\eta_0$ for selling immediately the non-liquid asset. Our aim is to find a dynamic choice of the amount of the reserve invested in the non-liquid asset which maximizes the survival probability, the company goes to ruin when the whole reserve becomes negative.
Let us call $x$ and $y$ the initial reserves in liquid asset and cash respectively, the associated Hamilton-Jacobi-Bellman equation of the optimization problem is

$$\max\{\mathcal{L}_1(\delta)(x,y), \mathcal{L}_2(\delta)(x,y)\} = 0 \text{ for } x \geq 0, y \geq 0,$$

where

$$\mathcal{L}_1(\delta)(x,y) = \delta_y(x,y) - \delta_x(x,y)$$

and

$$\begin{aligned} \mathcal{L}_2(\delta)(x,y) = \quad & (p + ry)\,\delta_x(x,y) - \beta\delta(x,y) + \beta \int_0^x \delta(x-\alpha, y)dF(\alpha) \\ & + \beta \int_x^{x+\frac{y}{1+\eta_0}} \delta(0, y - (1+\eta_0)\,(\alpha - x))dF(\alpha). \end{aligned}$$

We characterize the optimal survival probability $\delta$ as the unique viscosity solution of this equation with limit one at infinity.
This equation is a two-dimensional first-order non-linear integro-differential equation leading to a free-boundary problem. The optimal choice of the amount invested in the non-liquid asset and the amount kept in cash is specified by the so-called *region of inaction.* Inside the region of inaction, that is the set in which $\mathcal{L}_1(\delta) < 0$ and $\mathcal{L}_2(\delta) = 0$, the reserve invested in the non-liquid asset should not change, outside this region the management should invest immediately a positive amount of the reserve in the non-liquid asset.
For exponential claim-size distributions, we obtain the optimal survival probability and we found a closed-form expression for the boundary of the region of inaction.

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**Positivity, Local Smoothing Effects and Harnack Inequalities for Very Fast Diffusion Equation**

Matteo Bonforte

We investigate qualitative properties of local solutions $u(t,x) \geq 0$ to the fast diffusion equation, $\partial_t u = \Delta(u^m)/m$ with $m < 1$, and $\partial_t u = \Delta_p u = \nabla \cdot (|\nabla u|^{p-2}\nabla)$ with $1 < p < 2$, corresponding to general nonnegative initial data.
Our main results are quantitative positivity and boundedness estimates for locally defined solutions in domains of $[0,T] \times \mathbb{R}^d$. They combine into forms of new Harnack inequalities that are typical of fast diffusion equations.
Such results are new for $m$ and $p$ in the so-called very fast diffusion range, precisely for all $m \leq m_e = (d-2)/d$ , and/or for all $1 < p \leq 2d/(d+1)$, where $d$ is the dimension of the Euclidean space $\mathbb{R}^d$. In the supercritical case we recover the (sharp) results existing in literature with a different proof. The boundedness results are true even for $m \leq 0$, or $p = 1$, while the positivity ones cannot be true in that range.
For the fast $p$-Laplacian we also prove a new local energy inequality for suitable norms of the gradients of the solutions, which can be extended to more general operators of $p$-Laplacian type. As a consequence, we show that bounded local weak solutions are indeed local strong solutions for any $1 < p < 2$, more precisely $\partial_t u \in L^2_{loc}$.

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**Reaction diffusion equations with delay**

Carolina Capatto and Noemi Wolanski

Reaction diffusion equations have been used to model different types of problems in chemistry, physics and biology. Many processes depend on past states so delay differential equations are used.
On the other hand, it is natural to consider long range diffusions when modeling diffusive processes, specially in the case of biological or biomedical problems. In this case the diffusive term is no longer a differential operator but an integral one. It is a non–local operator.
We consider the following non–local differential equation with delay

$$\begin{cases} \dfrac{\partial u}{\partial t}(x,t) - J * u(x,t) + u(x,t) = g\left(\dfrac{1}{r}\displaystyle\int_{t-r}^{t} u(x,s)\ ds\right) & x \in \mathbb{R}^N, t > 0 \\ \\ u(x,s) = \phi(x,s) & x \in \mathbb{R}^N, -r \le s \le 0 \end{cases}$$

where $J : \mathbb{R}^N \to \mathbb{R}$ is non negative with integral one, $J(z) = J(-z)$, and the convolution is taken on the space variables. Moreover, $g \in C^1(\mathbb{R})$ and $g(0) = 0$.
We study local existence and uniqueness of a solution. Then, we investigate the possibility of finite time blow up. In particular, we prove that if the existence time $T$ is finite,

$$||\frac{1}{r}\int_{t-r}^{t} |u(x,s)|\, ds||_{L^\infty(\mathbb{R}^N)} + ||u(x,t)||_{L^\infty(\mathbb{R}^N)} \to\ +\infty \qquad \text{as } t \nearrow T.$$

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## An iterative method for a nonlinear two-point boundary value problem

Pedro Pablo Cárdenas Alzate

We study the semilinear second order ODE

$$u'' + g(t, u(t), u'(t)) = 0$$

under a nonlinear two-point boundary condition. An existence result is obtained by the method of upper an lower solutions. Moreover, we develop an iterative scheme that converges to a solution of the problem.
Joint work with Pablo Amster.

*Keywords*: Nonlinear two-point boundary conditions - upper and lower solutions - iterative methods

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**Some optimization problems for p-laplacian type equations**

Leandro Del Pezzo

In this work we study some optimization problems for nonlinear elastic membranes. More precisely, we consider the problem of optimizing the cost functional

$$\mathcal{J}(u) = \int_{\partial\Omega} f(x)u \,\mathrm{d}\mathcal{H}^{N-1}$$

over some admissible class of loads $f$, where $\mathcal{H}^d$ denotes the $d-$dimensional Hausdorff measure and $u$ is the (unique) solution to the nonlinear membrane problem with load $f$

$$\begin{cases} -\Delta_p u + |u|^{p-2}u = 0 & \text{in } \Omega, \\ |\nabla u|^{p-2}\frac{\partial u}{\partial \nu} = f & \text{on } \partial\Omega. \end{cases} \tag{1}$$

Here, $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is the usual $p-$Laplacian and $\frac{\partial}{\partial \nu}$ is the outer unit normal derivative.

In this work, we have chosen three different classes of admissible functions $\mathcal{A}$ to work with.

- The class of rearrangements of a given function $f_0$.
- The (unit) ball in some $L^q$.
- The class of characteristic functions of sets of given surface measure.

This latter case is what we believe is the most interesting one and where our main results are obtained.
For each of these classes, we prove existence of a maximizing load (in the respective class) and analyze properties of these maximizers.
When we work in the unit ball of $L^q$ the problem becomes trivial and we explicitly find the (unique) maximizer for $\mathcal{J}$, namely, the first eigenfunction of a Steklov-like nonlinear eigenvalue problem.
Finally we arrive at the main part of the paper, namely, the class of characteristic functions of sets of given boundary measure. In order to work within this class, we first relax the problem and work with the weak* closure of the characteristic functions (i.e. bounded functions of given $L^1$ norm), prove existence of a maximizer within this relaxed class and then prove that this optimizer is in fact a characteristic function. Then, in order to analyze properties of this maximizer, we compute the first variation (or shape derivative) with respect to perturbations on the set where the characteristic function is supported.
Joint work with J. Fernández Bonder.

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**Radial solutions of hamiltonian elliptic system with weights**

Irene Drelichman

In this poster we outline the main ideas contained in [4], where we prove the existence of infinitely many radial solutions in $\mathbb{R}^n$ for the weighted hamiltonian elliptic system

$$\begin{cases} -\Delta u + u &= |x|^a |v|^{p-2} v \\ -\Delta v + v &= |x|^b |u|^{q-2} u \end{cases} \tag{1}$$

under the hypotheses

$$p, q > 2,\ \frac{1}{p} + \frac{1}{q} < 1$$
$$0 < a < \frac{(n-1)(p-2)}{2}, \quad 0 < b < \frac{(n-1)(q-2)}{2}$$
$$\frac{n+a}{p} + \frac{n+b}{q} > n-2$$

and

$$q < \frac{2(n+b)}{n-4}, \quad p < \frac{2(n+a)}{n-4} \quad \text{if } n \geq 5$$

This result complements previously known existence results for unweighted systems of this kind in $\mathbb{R}^n$ [1] and for both weighted and unweighted systems on a bounded domain $\Omega$ with Dirichlet boundary conditions [2, 1, 3].
A key tool in our proof is a weighted imbedding theorem for fractional-order Sobolev spaces, that we present briefly.
We also outline how these results can be extended to a more general Hamiltonian elliptic system with weights under suitable hypotheses.
Joint work with P.L. De Nápoli and R.G. Durán.

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**Singular limits arising in periodic Chern-Simons vortices**

Pablo Figueroa and Manuel Del Pino

Consider the problem

$$
\begin{cases} -\Delta u & = \lambda e^u(1-e^u) - 4\pi(\delta_{p_1}+\delta_{p_2}), \quad \text{in } \Omega \\ & u \text{ doubly periodic on } \partial\Omega, \end{cases} \tag{1}
$$

where $\Omega = \{z = s\alpha + t\beta \in \mathbb{C} \mid 0 < s,t < 1\}$ with $\alpha, \beta \in \mathbb{C}\setminus\{0\}$, $\mathrm{Im}(\beta/\alpha) > 0$, $p_1, p_2 \in \Omega$ (not necessarily distinct), $\delta_p$ denotes a Dirac mass supported at $p$ and $\lambda > 0$. Using similar methods and transformations shown in [2, 3] and some intrinsic features of the problem shown in [5, 6], we have proved the existence of a family of solutions $\{u_\lambda\}_{\lambda>\lambda_0}$ (some $\lambda_0 > 32\pi/|\Omega|$) to (1) such that

$$
\lim_{\lambda\to+\infty} \lambda \int_\Omega e^{u_\lambda(x)}(1-e^{u_\lambda(x)})\,dx = 8\pi.
$$

Moreover, there is a point $q \in \Omega\backslash\{p_1,p_2\}$ satisfying $G(q,p_1)+G(q,p_2) = \min_{x\in\Omega}[G(x,p_1)+G(x,p_2)]$ such that $\lambda e^{u_\lambda}(1-e^{u_\lambda}) \rightharpoonup 8\pi\delta_q$ as $\lambda \to +\infty$ in measure sense, where $G$ is the Green's function of $-\Delta$ with respect to doubly periodic boundary conditions on $\partial\Omega$. This family of solutions turns out to be of non-topological-type [1, 6]. Problem (1) arises in the Chern-Simons vortex theory [4].

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**Blow-up for a discrete non-linear problem with non-local diffusion**

Alberto Forero Poveda

We study a discrete non-linear problem with non-local diffusion and a nonlinear source, namely

$$(u_i)'(t) = \sum_{j=-N}^{N} hJ\Big(\frac{h(i-j)}{u_j(t)}\Big) - = \sum_{j=-N}^{N} hJ\Big(\frac{h(i-j)}{u_i(t)}\Big) + f(u_i(t)), \tag{1}$$

together with an initial datum $u_i(0) = u_0(x_i) > 0$.
We show existence and uniqueness of s solution for $f$ Lipschitz using a fixed point argument in $C([0,t_0]; l^1_h)$, with the norm $|||w||| = \max_{\{0\le t\le t_0\}} \sum_{i=-N}^{N} h|w_i|(t)$. Moreover, we prove that there is a comparison principle for positive solutions.
We also study the existence of solutions that exhibit the blow-up phenomena, that is, there exists a finite time $T$ such that

$$\lim_{t\nearrow T} \max_j u_j(t) = +\infty.$$

We find that this happens if $f$ is positive, convex and verifies

$$\int_a^\infty \frac{1}{f(s)}\, ds < +\infty.$$

Finally we find the blow-up rates for certain specific functions $f$, for example, for $f(s) = s^p$ (with $p > 1$) and for $f(s) = e^s$.
For the proofs we use ideas from [4]. These results extend previous analysis done in [2]. See also [1] and [3] for related results.

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**Anaerobiosis in saturated soil aggregates**

Adriana González, Juan Carlos Reginato, and Domingo Alberto Tarzia.

A new model for the diffusion of oxygen in spherical aggregates of soils with simulta-neous consumption at constant, linear, and nonlinear rate is proposed. To evaluate the ef-fects of the dynamics of soil domain in which the diffusion and consumption takes place, a free-boundary formulation is studied. The three resultant free-boundary models are solved by a modification of the Constrained Integral Method. The obtained results are consistent with those presented by earlier authors who have solved the problems using fixed domains. The results obtained by our model for diffusion with constant consumption slightly overpredict the experimental values with respect to the results obtained by Currie (J Currie. Gaseous diffu-sion in the aeration of aggregated soil. Soil Sci. 1961;92:40-45) in fixed domains. Moreover, the results obtained by our model for diffusion with simultaneous nonlinear consumption overpredict the experimental values, partially improving the underpredicted results of Sierra et al. (J Sierra, P Renault, and V Valles. Anaerobiosis in saturated soil aggregates: modelling and experiment, Eur J Soil Sci. 1995;46:519-531) in fixed domains. Thus, our model could represent an alternative method to study and to generalize existing models.

Key words: Diffusion of oxygen, spherical aggregates of soils, free boundary problem, modified con-strained integral method.

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## Evolution of polyhedral crystal

Przemyslaw Górka

We consider a system modeling evolution of a single crystal grown from vapor. We account for vapor diffusion equation and Gibbs-Thomson relation on the crystal surface. We also assume that the velocity of the growing crystal is determined by the normal derivative of concentration of vapor at the surface, it is the so-called Stefan condition. We show local in time existence of solutions assuming that the initial crystal has admissible shape.

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**Time-space white noise eliminates global solutions in reaction-diffusion problems**

Pablo Groisman

We prove that perturbing the reactiondiffusion equation

$$u_t = u_{xx} + (u^+)^p \quad (p > 1),$$

with timespace white noise produces that solutions explodes with probability one for every initial datum, opposite to the deterministic model where a positive stationary solution exists.
Joint work with Julián Fernández Bonder.

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**Korn inequality and Divergence problem on domains with a cusp**

Fernando López García

If $\Omega \subset \mathbb{R}^n$ is a bounded domain, the existence of solutions $\mathbf{u} \in H_0^1(\Omega)^n$ of $\operatorname{div} \mathbf{u} = f$ for $f \in L^2(\Omega)$ with vanishing mean value, is a basic result in the analysis of the Stokes equations. Furthermore, this result has a strong relation with the Korn inequality which is fundamental in the analysis of the elasticity equation.
It is known that both mentioned results hold when $\Omega$ is a Lipschitz domain and that they are not valid if $\Omega$ presents external cusps.
In this poster we will show that for a class of domains with one cusp both results hold in weighted Sobolev spaces, where the weight is a power of the distance to the cusp. Moreover, this power is optimal.
Joint work with Ricardo Durán.

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**An extension of a Nirenberg result applied to the 2-body problem**

Manuel Maurette

We recall the perturbed 2-body problem:

$$(1) \qquad \begin{cases} x'' \pm \frac{x-y}{|x-y|^3} = p_1(t) & t \in (0,T) \\ y'' \pm \frac{y-x}{|x-y|^3} = p_2(t) & t \in (0,T) \\ (x,y)(0) = (x,y)(T), & (x,y)'(0) = (x,y)'(T) \end{cases}$$

It can be easily taken to a central-motion problem by a simple transformation, nonetheless, we attack the equation differently, exposing the intrinsic difficulties of the problem. We do so by using topological degree techniques, extending a result from Nirenberg [2] and ideas from the works of Ortega-Ward [3], and Amster-De Nápoli [1].

We study (1) in the context of a nonlinear periodic system of second order differential equations:

$$(2) \qquad \begin{cases} u'' = p(t) - g(u,u') & 0 < t < T \\ u(0) = u(T), & u'(0) = u'(T) \end{cases}$$

Where $p$ is a continuous function with $\overline{p} = 0$ and the nonlinearity $g$ is continuous and is bounded. Problems like (2) are said to be at resonance.

The case when the nonlinearity is of the form $cu' + g(u)$, a classical result from Nirenberg [2] assures the existence of solutions when $g$ has uniform radial limits $g_v = \lim_{s\to+\infty} g(sv)$ for $v \in S^{N-1}$, supposing that $g_v \neq \overline{p}$ for all $v$, and that the degree of the application $\frac{g_v - \overline{p}}{|g_v - \overline{p}|}$ is not null.

In [3], this result is extended, for the case $\overline{p} = 0$, allowing the nonlinearity to vanish at infinity, which is the case in the 2-body problem. In [1] Nirenberg's radial condition is generalized by a weaker one, in which radial limits of $g$ are not required.
Joint work with Pablo Amster.

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**Sobolev Interior Regularity for Weak Solutions of Quasilinear Degenerate Elliptic Equations**

C. Gariboldi, G. Giubergia and F. Mazzone

In this work we consider weak solutions of an equation of the following kind

$$\operatorname{div} A(x, \nabla u(x)) = g(x), \tag{1}$$

where the function $A = (a^1, ..., a^n) \in C^0(B_r \times \mathbb{R}^n) \cap C^1(B_r \times \mathbb{R}^n - \{0\})$ satisfies the structures conditions

$$c_1 w(x)(1+|\eta|)^{p-2}|\xi|^2 \leq \frac{\partial a^i}{\partial \eta_j}\xi_i\xi_j \tag{2}$$

$$|a^i| + \left|\frac{\partial a^i}{\partial x_j}\right| + (1+|\eta|)\left|\frac{\partial a^i}{\partial \eta_j}\right| \leq c_3 w(x)(1+|\eta|)^{p-1} \quad \text{for } i,j = 1, ..., n \tag{3}$$

Here $w$ denotes a weight function, i.e. $w$ is a nonnegative locally integrable function on $\mathbb{R}^n$. Moreover it is assumed that w is in the Muckenhoupt class $A_1$ uniformly in each coordinate. On the function $g$ we suppose that $g/w \in L^2(B_r, w)$.
We deal with a function $u \in W^{1,p}(B_r, w)$ ($W^{1,p}(B_r, w)$ denotes a Sobolev weighted space) which is a weak solution of (1). In this work we obtain Sobolev estimates for $u$, more specifically we prove

**Theorem 1.** Let $u \in W^{1,p}(B_r, w)$, $p \geq 2$, be a weak solution of (1). We assume that $A$ satisfies the structure conditions (2) and (3), $g/w \in L^2(B_r, w)$ and $w$ uniformly in $A_1$ w.r.t each coordinate. Then $u \in W^{2,2}(B_{\frac{r}{2}}, w)$ and the second derivatives of $u$ satisfy that

$$\int_{B_{\frac{r}{2}}} |D_{ij}u|^2(1+|\nabla u|)^{p-2}\,dx < \infty \quad i,j = 1, ..., n.$$

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**Lyapunov inequalities in $\mathbb{R}^n$**

Juan Pablo Pinasco

The Lyapunov inequality has many applications in different areas of ordinary differential equations, like stability problems, oscillation theory, a priori estimates, inequalities, and eigenvalue bounds. Different proofs and extensions of this inequality have been appeared in the literature, always for the one dimensional problem. However, there are few works devoted to similar inequalities for partial differential equations in $\mathbb{R}^N$. We derive a Lyapunov type inequality for the problem

$$\begin{cases} \Delta_p u + q(x)|u|^{p-2}u = 0, & x \in \Omega \\ u(x) = 0, & x \in \partial\Omega \end{cases} \tag{1}$$

where $u \in W_0^{1,p}(\Omega)$, and $p > N$, involving the inner radius of $\Omega$ and the $\|q\|_1$.
For $p \leq N$ we show that similar inequalities cannot hold, and certain Lyapunov inequalities involving $\|q\|_s$ hold only for $s > N/p$.
We derive eigenvalue bounds from them, improving some of the previous known estimates.
Joint work with Pablo de Nápoli.

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**Refined asymptotics for eigenvalues of the p-Laplace operator in one space dimension**

Ariel Salort

In this work we study the spectral counting function for the $p$-Laplace operator in one dimension. We show the existence of a two term Weyl-type asymptotic. The method of proof is rather elementary, based on the Dirichlet lattice points problem, which enable us to obtain similar results for domains of infinite measure.

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**Multiple solutions for the $p-$laplace operator with critical growth**

Analía Silva

In this poster we show the existence of at least three nontrivial solutions to the following quasilinear elliptic equation $-\Delta_p u = |u|^{p^*-2}u + \lambda f(x,u)$ in a smooth bounded domain $\Omega$ of $\mathbb{R}^N$ with homogeneous Dirichlet boundary conditions on $\partial\Omega$. The proof is based on variational arguments and the classical concentrated compactness method.
Joint work with Pablo De Nápoli and Julián Fernández Bonder.

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**Flatness results for elliptic problems on manifolds and of non local types**

Yannick Sire

I will describe several flatness results which can obtained through different techniques for elliptic PDEs on Riemannian manifolds and PDE's involving non local operators. These results generalize De Giorgi conjecture in these frameworks. These results show that under natural assumptions on entire solutions, at least in 2D, these solutions are one-dimensional. I will also describe some results for boundary reaction equations, of semi-linear and quasi-linear types.

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**Nonlinear problems involving the square root of the laplacian**

Jinggang Tan

We consider nonlinear elliptic problems involving a nonlocal operator: the square root of the Laplacian in a bounded domain with zero Dirichlet boundary conditions. We establish the existence of positive solutions for problems with power non- linearities in the subcritical case, Brézis-Nirenberg type existence results for the critical problems under a small perturbation, non- existence of positive solutions in some supercritical problems. We also present the regularity and an $L^\infty$ estimate of Brezis-Kato type for weak solutions, nonlinear Liouville type results, a priori esti mates of Gidas-Spruck type and a symmetry result of Gidas-Ni- Nirenberg type.
Joint work with Xavier Cabré.

Key words and phrases. Fractional Laplacian, variational methods, a priori es timates, moving planes method.

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**Qualitative properties of saddle solutions to bistable diffusion equations**

Joana Terra

In a joint work with Xavier Cabré we consider saddle-shaped solutions of the semilinear elliptic equation $-\Delta u = f(u)$ in the whole $\mathbb{R}^{2m}$, where $f$ is of bistable type. It is known that there exists a saddle-shaped solution in $\mathbb{R}^{2m}$. This is a solution which changes sign in $\mathbb{R}^{2m}$ and vanishes only on the Simons cone $\mathcal{C} = \{(x^1, x^2) \in \mathbb{R}^m \times \mathbb{R}^m : |x^1| = |x^2|\}$. It is also known that this solution is unstable in dimensions 2 and 4.
Here we establish two main results. The first one concerns the asympotic behaviour of solutions vanishing on $\mathcal{C}$ which are odd with respect to $\mathcal{C}$ and positive in the region $\{(x^1, x^2) \in \mathbb{R}^m \times \mathbb{R}^m : |x^1| > |x^2|\}$. The second one establishes that in the case where $2m = 6$ every such solution, in particular every saddle-shaped solution, is unstable outside of every compact set and, as a consequence has finite Morse index. Moreover we prove the existence of a minimal and a maximal saddle-shaped solution and derive monotocitiy properties for the maximal solution.
These results are relevant in connection with a conjecture of De Giorgi extensively studied in recent years.

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**A priori estimates of signed-solutions for nonlinear elliptic problems and applications**

Carlos Vélez

We study the boundary value problem

$$\Delta u + f(u) = 0 \quad \text{in } \Omega, \quad u = 0 \quad \text{on } \partial\Omega, \tag{1}$$

where $\Omega \subseteq \mathbb{R}^N$, $N \geq 3$, is a bounded smooth domain and $f : \mathbb{R} \to \mathbb{R}$ is a $C^1-$ nonlinear function. Under suitable conditions, in particular when f is sublinear, we establish a priori estimates for positive (and negative) solutions of (1) by following some of the ideas used by De Figueiredo, Lions and Nussbaum in the superlinear case. In our setting, however, the estimates are more precise and allow us to prove existence and qualitative property results for asymptotically linear problems. In particular, the existence of signchanging solutions with a *large* Morse index can be proved under additional conditions and using nonlinear analysis techniques.
To be more precise, let $\epsilon > 0$ and assume $f$ satisfies:

$$f(0) = 0, \tag{E1}$$

$$\text{there exists } D_f \in (0, \infty) \text{ such that } f'(t) \leq D_f, \text{ for all } t \in \mathbb{R}, \tag{E2}$$

$$\text{there exists } A > 0 \text{ such that } f'(t) \geq \lambda_1 + \epsilon, \text{ for all } , |t| > A, \tag{E3}$$

where $\lambda_1$ is the first eigenvalue of $-\Delta$ with zero Dirichlet boundary condition. Lets define $K_f, m_f$ and $M_f$ by

$$-m_f := \min_{t \geq 0} f(t), -M_f := \min_{t \geq 0}\{f(t) - (\lambda_1 + \epsilon)t\}, \ -K_f := \min_{t \geq 0}\{t|t|^{\frac{1}{N+1}} - f(t)\}.$$

Our result reads as follows.

**Theorem 1.** Let $f : \mathbb{R} \to \mathbb{R}$ be a $C^1-$function satisfying $(E1)$, $(E2)$, and $(E3)$. There exist positive constants $C_i(\Omega, N)$, $i = 1, ..., 5$, and $r_f > 0$, depending only of $\Omega, N, \|f'\|_{L^\infty(\mathbb{R})}$ and $D_f$ such that if $u$ is a positive solution of (1) then

$$\|u\|_{L^\infty(\Omega)} \leq M_f^{1+\frac{1}{N+1}} r_f^{2-N} C_1(\Omega, N) + K_f r_f^{2-N} C_2(\Omega, N) + m_f C_3(\Omega, N)$$

and

$$\|u\|_{L^\infty(\Omega)} \leq M_f D_f r_f^{2-N} C_4(\Omega, N) + m_f C_5(\Omega, N)$$

By combining these estimates with some abstract nonlinear techniques, the existence of sign-changing solutions can be established. As an example, we show the following result.

**Theorem 2.** Let $f : \mathbb{R} \to \mathbb{R}$ be a $C^1-$function satisfying $(E1)$, and

$$f'(\infty) := \lim_{|t| \to \infty} f'(t) \in (\lambda_k, \lambda k + 1) \text{ for } k \geq 2, \tag{E4}$$

where $\lambda_k$ is the $k$-th eigenvalue of $-\Delta$ with zero Dirichlet boundary condition.
There exists a positive constant $C(\Omega, N, f)$ depending on $\Omega, N$ and $f$ with the following property: If

$$f'(t) < \lambda_k \quad \forall t \in [-C(\Omega, N, f), C(\Omega, N, f)],$$

Then there exists at least a sign-changing solution $u_*$ of (1) such that

$$\|u_*\|_{L^\infty(\Omega)} > C(\Omega, N, f).$$

Joint work with Alfonso Castro and Jorge Cossio.

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