Harmonic Analysis, Fractal Geometry and Applications

A Conference in Honor of Ursula Molter On her 60th Birthday

UM2017 - August 4, 14:40 – 14:50

Introduction

Noemí Wolanski
Universidad de Buenos Aires, Argentina

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Wavelets on the hunt for gravitational waves

Stéphane Jaffard
Université Paris-Est Créteil Val de Marne, France

On September 14th 2015, LIGO (Laser Interferometer Gravitational-Wave Observatory) in USA, performed the first detection of a gravitational wave generated by the coalescence of two black holes. The signal processing algorithm which allowed this detection uses in a crucial way a variant of wavelet bases called "Wilson bases" (it is an orthonormal "time-frequency" decomposition, as opposed to standard wavelets which are of "time-scale" type). We will mention the origin of such basses, which starts with the seminal work of Gabor in the 50s, and was made more precise by K. Wilson at the beginning of the 80s (motivated by renormalization theory). We will then show why such bases are particularly well adapted to such gravitational waves, and which technical choices were made in the detection algorithm. Finally, we will mention the perspectives opened by this new type of astronomy which, for the first time, is not based on light or electromagnetic waves detection, and the role that such bases, or variants are expected to play in it.

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Greedy algorithm and embeddings

Eugenio Hernández
Universidad Autónoma de Madrid, Spain

The greedy algorithm is a way to approximate elements of a Banach space by using the biggest coefficients of the representation of the element in a given basis. We will show how to obtain general embeddings between a Banach space and weighted Lorentz spaces and use them to quantify how good is this algorithm in comparison with the best approximation. Several examples will be presented. This is a joint work with P. Berná, O. Blasco, G. Garrigós and T. Oikhberg.

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Wavelets, Self-Similarity, and the Joint Spectral Radius: A Retrospective

Christopher Heil
Georgia Institute of Technology, USA

Three decades ago, the "Wavelet Revolution" unexpectedly brought us the construction wavelet orthonormal bases generated by smooth functions. Central to these constructions is the theory of multiresolution analyses and the existence of refinable *scaling functions*, whose graphs exhibit a certain type of Self-similarity. In this talk we will examine wavelets, refinablity, and self-similarity from the point of view of the *joint spectral radius* of families of matrices, following the work of Cabrelli and Molter over time in the development of generalized self-similarity, multiwavelets, and multiwavelets in higher dimensions.

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Weighted endpoint estimates for commutators of Calderón-Zygmund operators.

Sheldy Ombrosi
Universidad Nacional del Sur, Argentina

In this talk we will show recent advances in borderline weighted estimates for the Coifman, Rochberg and Weiss commutator. Given a symbol $b \in \mathbf{BMO}$ and a Calderón-Zygmund operator T we study two-weight estimates for the commutator $[b, T]$. We also obtain quantitative estimates in the case of the weights satisfy certain additional geometric conditions. This talk is based in a joint work with Andrei Lerner and Israel Rivera-Ríos.

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Improved Poincaré inequalities in fractional Sobolev spaces.

Ricardo Durán
Universidad de Buenos Aires , Argentina

First we recall the classic Poincaré and Sobolev-Poincaré inequalities for functions with vanishing mean value in a bounded domain. Then we present the so-called improved versions of these inequalities and mention some important consequences.

Finally, we present generalizations of these inequalities for functions in fractional order spaces for the cases of John, s-John, and Hölder α domains, and we discuss the optimality of our results.

The results presented were obtained in collaboration with Irene Drelichman.

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Frames and some algebraic forays

John Benedetto
University of Maryland, USA

We formulate a mathematical theory of frame multiplication, in which two essential algebraic operations can be made compatible in a natural way. The motivation comes from our approach to defining vector-valued ambiguity functions. These are formulated to provide realistic modelling of multi-sensor environments in which a useful time-frequency analysis is required, and they depend on the construction of finite number-theoretic sequences that have constant amplitude and zero-autocorrelation, i.e., CAZAC sequences.

The functions whose ambiguity function we wish to define have a given finite group G as their domain. Our results have the following form: i. if frame multiplication exists in the context of the aforementioned operations, then the vector-valued ambiguity function is well-defined; ii. frame multiplication exists if and only if the finite frames that arise in the theory are of a certain type, e.g., harmonic frames or, more generally, group frames.

For infinite locally compact abelian groups (LCAGs) G, we focus on those that are central in generalizations of p-adic number theory, i.e., G will have compact open subgroups H. We prove the analogue in this setting of the fact, true on Euclidean space and all similar LCAGs, in which frames of translates of f are characterized in terms of zero sets of periodizations of the Fourier transform of f. In the compact open subgroup case, there is a roadblock at the outset, and translation has to be correctly defined.

The frame multiplication theory is a collaboration with Travis Andrews and Jeffrey Donatelli. The earlier CAZAC work (2012), which is only a backdrop to introduce the setting of the talk, was a collaboration with Robert Benedetto and Joseph Woodward. The frames of translates result proved with Robert Benedetto, is essentially non-Euclidean in setting, but was guided by the analogous Euclidean theorem proved with Shidong Li in 1992.

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A problem of interacting obstacles

Luis Caffarelli
University of Texas, USA

Originally considered by Chipot and Vergara-Caffarelli this family of problems concerns "deformable" functions, pressing against each other like an elastic body against a membrane or the income and cost of an asset We will discuss the existence and geometry of solutions

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Existence and uniqueness theory for binary collisional kinetic models

Irena Martinez Gamba
University of Texas, USA

We focus on Chapman-Kolmogorov type equations for kinetic evolution models binary interactions and develop an existence and uniqueness theory by posing the problems as solving an ODE in a suitable Banach space. These models range from the classical Boltzmann equation for rarefied elastic gases with hard potentials as transition probability rates, to quantum Boltzmann models for Bose-Einstein Condensates (BEC) at very low temperature, to wave-turbulence models in stratified flows such as the Zakharov equation.

We show they share a common framework as being evolution problems for non-local multi-linear flows, whose solutions are continuous probability densities. There natural solutions spaces are those of observables, that is, integrable functions with polynomial weights (i.e. finite polynomial moments or expectations).

We show that solutions are constructed by means of solving ODEs in a convex, bounded subspace of positive functions in the Banach space with suitable weighted integrable functions, where polynomial moments estimates and interpolation tools are enough to show that a Holder, sub-tangent and one-sided Lipschitz conditions hold.

This work is in collaboration with Ricardo Alonso, Leslie Smith and Minh Binh Tran.

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Local Calderón-Zygmund theory on proper open subsets

Eleonor Harboure
IMAL - CONICET, Argentina

Inspired on the local Hardy-Littlewood and the Hilbert transform on $\Omega = (0, \infty)$, a proper open subset of $\mathbb{R}$, we develop a local Calderón-Zygmund theory in a quite general context. We start with (X, d), a metric space having the PHD property, a purely geometric condition, and a proper open subset $\Omega \subset X$. We further assume that a Borel measure is defined in Ω having the doubling property only for a family of balls that "stay away" from $\partial\Omega$. We construct a Hardy-Littlewood maximal operator on this family and establish a theory of weights for the L^p-spaces. We introduce also the corresponding singular integrals that, to some extent, are controlled by the maximal function. All these operators are local in the sense that when applied to a function f, their values at a point x only depend on the values of f on some neighborhood of x. These results are part of joint work with O. Salinas and B. Viviani.

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Who is Ursula Molter?

Akram Aldroubi
Vanderbilt University, USA
