Existence and uniqueness theory for binary collisional kinetic models

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We focus on Chapman-Kolmogorov type equations for kinetic evolution models binary interactions and develop an existence and uniqueness theory by posing the problems as solving an ODE in a suitable Banach space. These models range from the classical Boltzmann equation for rarefied elastic gases with hard potentials as transition probability rates, to quantum Boltzmann models for Bose-Einstein Condensates (BEC) at very low temperature, to wave-turbulence models in stratified flows such as the Zakharov equation.

We show they share a common framework as being evolution problems for non-local multi-linear flows, whose solutions are continuous probability densities. There natural solutions spaces are those of observables, that is, integrable functions with polynomial weights (i.e. finite polynomial moments or expectations).

We show that solutions are constructed by means of solving ODEs in a convex, bounded subspace of positive functions in the Banach space with suitable weighted integrable functions, where polynomial moments estimates and interpolation tools are enough to show that a Holder, sub-tangent and one-sided Lipschitz conditions hold.

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