Local Calderón-Zygmund theory on proper open subsets

Eleonor Harboure
IMAL - CONICET, Argentina

Inspired on the local Hardy-Littlewood and the Hilbert transform on $\Omega = (0, \infty)$, a proper open subset of $\mathbb{R}$, we develop a local Calderón-Zygmund theory in a quite general context. We start with (X, d), a metric space having the PHD property, a purely geometric condition, and a proper open subset $\Omega \subset X$. We further assume that a Borel measure is defined in Ω having the doubling property only for a family of balls that "stay away" from $\partial\Omega$. We construct a Hardy-Littlewood maximal operator on this family and establish a theory of weights for the L^p-spaces. We introduce also the corresponding singular integrals that, to some extent, are controlled by the maximal function. All these operators are local in the sense that when applied to a function f, their values at a point x only depend on the values of f on some neighborhood of x. These results are part of joint work with O. Salinas and B. Viviani.