Super-resolution by means of Beurling minimal extrapolation

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We address the super-resolution question: Given spectral data defined on a finite set of d-dimensional multi-integers; of all complex Radon measures on the d-dimensional torus, whose Fourier transform equals this data, does there exist exactly one with minimal total variation? We first note that this is a mathematical formulation of a large class of super-resolution problems that arises in image processing, that it generalizes some fundamental problems in compressed sensing, and that it has wide ranging applications in other fields.

We prove a theorem that has quantitative implications about the possibility and impossibility of constructing such a unique measure. Our method introduces the notion of an admissibility range that fundamentally connects Beurling's theory of minimal extrapolation with the Candes and Fernandez-Granda theory of super-resolution. The method is also well -suite for the construction of explicit examples.

This is part of an on-going collaboration with Weilin Li.