X Encuentro Nacional de Analistas A.P. Calderón
La Falda (Córdoba), 25 al 28 de Agosto de 2010

Best simultaneous monotone approximants in Orlicz spaces

Fabián Eduardo Levis [1]

Abstract

Let $\mathcal{M}_0$ be the class of all real extended μ-measurable functions on $[0,1]$, where μ is the Lebesgue measure.

Let $\phi : \mathbf{R}_+ \to \mathbf{R}_+$ be a differentiable and convex function, $\phi(0) = 0$, $\phi(t) > 0$, $t > 0$. For $f \in \mathcal{M}_0$, let

$$\Psi_\phi(f) := \int_0^1 \phi(|f(x)|)d\mu(x).$$

Several authors studied geometric properties of the Orlicz space

$$L_\phi[0,1] := \{f \in \mathcal{M}_0 : \Psi_\phi(\lambda f) < \infty \ \text{ for some } \ \lambda > 0\}.$$

Under the Luxemburg norm, $L_\phi[0,1] =: L_\phi$, is a Banach space. It is easy to see that if $\phi(t) = t^p$, $1 \leq p < \infty$, we obtain the Lebesgue space L_p and $\Psi_\phi(f) = \|f\|_p^p$.

We assume that ϕ satisfies the Δ_2-condition, i.e., there exists $K > 0$ such that $\phi(2t) \leq K\phi(t)$ for all $t \geq 0$. So,

$$L_\phi = \{f \in \mathcal{M}_0 : \Psi_\phi(\lambda f) < \infty \ \text{ for all } \ \lambda > 0\}.$$

Given $\mathcal{D} \subset L_\phi$ and $f^j \in L_\phi$, $1 \leq j \leq m$, we consider the problem of finding $g \in \mathcal{D}$ such that

$$\sum_{j=1}^{m} \Psi_\phi(f^j - g)w_j = \operatorname*{ínf}_{h\in\mathcal{D}} \sum_{j=1}^{m} \Psi_\phi(f^j - h)w_j, \tag{1}$$

where w_j are positive weights. Denote $\mathcal{M}_{\phi,\mathbf{w}}(\mathbf{f},\mathcal{D})$ the set of elements $g \in \mathcal{D}$ verifying (1), where $\mathbf{f} = (f^1, ..., f^m)$ and $\mathbf{w} = (w_1, ..., w_m)$. Each element of $\mathcal{M}_{\phi,\mathbf{w}}(\mathbf{f},\mathcal{D})$ is called a *best simultaneous approximant to* f^j, $1 \leq j \leq m$, *from* $\mathcal{D}$.

In this work, we give a characterization of best simultaneous approximants when $\mathcal{D}$ is the cone of nondecreasing left-continuous functions in L_ϕ. Moreover, we show an explicit formula from calculate of $\text{mín}\,\mathcal{M}_{\phi,\mathbf{w}}(\mathbf{f},\mathcal{D})$ and $\text{máx}\,\mathcal{M}_{\phi,\mathbf{w}}(\mathbf{f},\mathcal{D})$. Finally, we discuss the continuity of a best simultaneous monotone approximant when f^j, $1 \leq j \leq m$, are approximately continuous.

[1]Departamento de Matemática, FCEFQyN, Univ. Nac. de Río Cuarto,
Ruta 36 Km 601, 5800, Río Cuarto, Argentina. E-mail: flevis@exa.unrc.edu.ar