Around the A_2 conjecture for singular integrals and commutators

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We plan to survey on some recent result concerning the boundedness of singular integrals and commutators on weighted L^2 spaces with sharp bounds.

In the first part of this lecture (joint work with D. Cruz-Uribe and J. M. Martell) we will discuss a new proof of the linear sharp weighted L^2 estimate

$$\|T\|_{L^2(w)} \leq c_{n,T}\,[w]_{A_2} \tag{1}$$

where T is the Hilbert transform, a Riesz transform, the Beurling-Ahlfors operator or any operator that can be approximated by Haar shift operators which avoids the Bellman function technique and any two weight norm inequalities. The method can be applied to obtain similar sharp results for other important operators such as the dyadic square function and the vector-valued maximal operator.

In the second part (joint work with S. Treil and A. Volberg) of the lecture we will discuss some recent progress of the A_2 conjecture for any Calderón-Zygmund operator. In particular we show that everything is reduced to consider the corresponding weak L^2 estimate.

In the last part of the lecture (joint work with D. Chung and C. Pereyra) we will discuss different arguments to derive quadratic sharp estimates of the form

$$\|[b,T]\|_{L^2(w)} \leq c_{n,T}\,\|b\|_{BMO}\,[w]_{A_2}^2$$

where b is any BMO function and T is any linear operator satisfying (1). The novelty here is the quadratic exponent which is sharp.

In any of the above examples the L^p result is a consequence of the sharp extrapolation theorem.