Easy polynomials which are hard to interpolate

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Abstract

In this talk we introduce and discuss a new computational model for Hermite–Lagrange interpolation with non-linear classes of polynomial interpolants. We distinguish between an interpolation problem and an algorithm that solves it. Our model includes also coalescence phenomena and captures a large variety of known Lagrange–Hermite interpolation problems and algorithms. Like in traditional Hermite–Lagrange interpolation, our model is based on the execution of arithmetic operations (including divisions) in the field where the data (nodes and values) are interpreted and arithmetic operations are counted at unit costs. This leads us to a new view of rational functions and maps defined on arbitrary constructible subsets of complex affine spaces. For this purpose we have to develop new tools in algebraic geometry which themselves are mainly based on Zariski's Main Theorem and the theory of places (or equivalently: valuations). We finish this talk by exhibiting two examples of Lagrange interpolation problems with non-linear classes of interpolants, which do not admit efficient interpolation algorithms (one of these interpolation problems requires even an exponential quantity of arithmetic operations in terms of the number of the given nodes in order to represent some of the interpolants). In other words, classic Lagrange interpolation algorithms are asymptotically optimal for the solution of these selected interpolation problems and nothing is gained by allowing interpolation algorithms and interpolation classes to be non–linear. We show also that classic Lagrange interpolation algorithms are almost optimal for generic nodes and values. This generic data cannot be substantially compressed by using non-linear techniques.

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