Harmonic Analysis related to Schrödinger operators

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Let us consider the Schrödinger operator on $\mathbb{R}^d$, $d \geq 3$,

$$\mathcal{L} = -\Delta + V,$$

where the potential $V \geq 0$ is a function satisfying, for some $q > \frac{d}{2}$, the reverse Hölder inequality

$$\left(\frac{1}{|B|} \int_B V(y)^q \, dy \right)^{1/q} \leq \frac{C}{|B|} \int_B V(y) \, dy$$

for every ball $B \subset \mathbb{R}^d$.

The general theory of semigroups, in particular Yosida's generating Theorem, implies that $\mathcal{L}$ is the infinitesimal operator of a semigroup, formally denoted by $T_t = e^{-t\mathcal{L}}$, that solves the diffusion problem

$$\begin{aligned} \frac{d}{dt} u(\cdot, t) &= -\mathcal{L} u(\cdot, t), \\ u(\cdot, 0) &= f, \end{aligned}$$

by setting $u(x,t) = e^{-t\mathcal{L}} f(x)$.

In this talk we will introduce the main operators of the Harmonic Analysis in this context and we will make a review of their behavior on the L^p spaces, pointing out the similarities and the differences with the classical versions corresponding to the Laplacian.

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