REGULARITY OF SOME ELLIPTIC EQUATIONS

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Abstract. We begin by considering a bounded smooth domain $\Omega \subset \mathbb{R}^n$ and a non-negative function u, solution to a simple semi-linear elliptic problem

$$-\Delta u = \lambda g(u),$$

where g is a superlinear nondecreasing C^1 non-linearity. It in known there exists an extremal parameter $\lambda^* > 0$ such that for $\lambda > \lambda^*$ there exists no classical solution, for $0 \leq \lambda < \lambda^*$ there exists a minimal semi-stable classical solutions, and for $\lambda = \lambda^*$ there exists a weak solution. Can this extremal solution be regular, at least for certain dimensions n, functions g and domains Ω? Can you then perturb the equation to a non-variational setting such as

$$-\Delta u - b(x)|\nabla u|^2 = \lambda f(u)?$$

We provide some answers to this problem, involving careful estimates.

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