Geometric regularity estimates for quasilinear evolution models

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Introduction

In this Lecture we are interested in studying quantitative features for evolution models of $p-$Laplacian type as follows

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In this Lecture we are interested in studying quantitative features for evolution models of $p-$Laplacian type as follows

$$\mathcal{Q}u := \frac{\partial u}{\partial t} - \operatorname{div}(|\nabla u|^{p-2}\nabla u) = f(x,t) \quad \text{in} \quad \Omega_T, \quad p > 2 \tag{1.1}$$

where

- ✓ $\Omega_T := \Omega \times (0,T)$ with $\Omega \subset \mathbb{R}^N$ a bounded and regular domain;
- ✓ $f \in L^{q,r}(\Omega_T)$ (a Lebesgue space with mixed norms) endowed with the norm

$$\|f\|_{L^{q,r}(\Omega_T)} := \left(\int_0^T \left(\int_\Omega |f(x,t)|^q dx\right)^{\frac{r}{q}} dt\right)^{\frac{1}{r}} \text{[a]}.$$

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[a]

A. Benedek & R. Panzone, *The space L^p, with mixed norm*. Duke Math. J. 28 1961 301-324.

Motivation

A fundamental issue in linear and nonlinear PDEs consists in inferring which is the expected regularity to weak solutions.

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By way of motivation, let us visit the linear theory: Let u be a weak solution to:

$$\mathcal{H}u := \frac{\partial u}{\partial t}(x,t) - \Delta u(x,t) = f \text{ in } \quad Q_1^- := B_1 \times (-1,0]. \tag{1.2}$$

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There are two important aspects which we must take into account:

A priori estimate to Hom. problem with "frozen" coef. **Vs** Integrability of the source term

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Indeed, $v(x,t) := \frac{u(\rho x, \rho^2 t)}{\rho^\kappa}, \quad \kappa \in (0,2]$ verifies in the weak sense:

$$\frac{\partial v}{\partial t}(x,t) - \Delta v(x,t) = \rho^{2-\kappa} f(\rho x, \rho^2 t) := f_\rho(x,t) \quad \Rightarrow \quad \|f_\rho\|_{L^{q,r}(Q_1^-)} \leq \rho^{2-\kappa-\left(\frac{n}{q}+\frac{2}{r}\right)} \|f\|_{L^{q,r}(Q_1^-)}.$$

Sharp regularity estimates

More integrability of f $\Rightarrow$ More (local) regularity of u

Theorem (da S. and Teixeira, Math. Ann. 18)

Let u be a bounded weak solution to (1.2) then

$\mathbf{f \in L^{q,r}(Q_1^-)}$	**Sharp Regularity**
$1 < \frac{n}{q} + \frac{2}{r} < 2$	$C_{loc}^{\zeta, \frac{\zeta}{2}}(Q_1^-)$
$\frac{n}{q} + \frac{2}{r} = 1$	$C_{loc}^{0,Log\text{-}Lip}(Q_1^-)$
$0 < \frac{n}{q} + \frac{2}{r} < 1$	$C_{loc}^{1+\zeta, \frac{1+\zeta}{2}}(Q_1^-)$
$BMO \supset L^{\infty,\infty} \simeq L^{\infty}$	$C_{loc}^{1,Log\text{-}Lip}(Q_1^-)$

Explicit representation of the moduli of continuity

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$\mathbf{f \in L^{q,r}(Q_1^-)}$	**Sharp Regularity**
$1 < \frac{n}{q} + \frac{2}{r} < 2$	$C_{loc}^{\varsigma, \frac{\varsigma}{2}}(Q_1^-)$
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$$\varsigma := 2 - \left(\frac{n}{q} + \frac{2}{r}\right) \quad \textit{and} \quad \zeta := \min\left\{\alpha_{\text{Hom}}^-, 1 - \left(\frac{n}{q} + \frac{2}{r}\right)\right\}$$

Sharp Lipschitz Logarithmical moduli of continuity

Theorem (da S. and Teixeira, Math. Ann. 18)

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$\mathbf{f} \in \mathbf{L}^{\mathbf{q},\mathbf{r}}(\mathbf{Q}_1^-)$	**Sharp Regularity**
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$$\tau(s) := s \log s^{-1} \quad \text{and} \quad \psi(r) := s^2 \log s^{-1}$$

Motivation

One Million Dollar Question:

What should we expect from Nonlinear Scenery ($p \neq 2$)?

Recently, under the condition : $\frac{1}{r} + \frac{n}{pq} < 1 < \frac{2}{r} + \frac{n}{q}$ for $p > 2$ and by combining geometric tangential methods and intrinsic scaling techniques (cf. [5]), the sharp (geometric) $C^{\alpha, \frac{\alpha}{\theta}}_{\text{loc}}$ regularity estimate was established in Teixeira-Urbano[a], where

$$\alpha = \frac{p\left[1 - \left(\frac{1}{r} + \frac{n}{pq}\right)\right]}{p\left[1 - \left(\frac{1}{r} + \frac{n}{pq}\right)\right] + \left(\frac{2}{r} + \frac{n}{q}\right) - 1} \quad \text{and} \quad \theta := 2\alpha + (1-\alpha)p.$$

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E.V. Teixeira & J.M. Urbano, *A geometric tangential approach to sharp regularity for degenerate evolution equations.* **Anal. PDE** 7 (2014), no. 3, 733-744.

Motivation

One Million Dollar Question:

What should we expect from Nonlinear Scenery ($p \neq 2$)?

Essentially, Teixeira and Urbano leave as open issues the following scenarios:

$\mathbf{f} \in \mathbf{L}^{\mathbf{q},\mathbf{r}}(\mathbf{Q}_1^-)$	**Sharp Regularity**
$\frac{1}{r} + \frac{n}{pq} < 1$ and $1 < \frac{2}{r} + \frac{n}{q}$	$C_{loc}^{\alpha, \frac{\alpha}{\theta}}$
$\frac{1}{r} + \frac{n}{pq} < 1$ and $1 = \frac{2}{r} + \frac{n}{q}$	Open Problem
$0 < \frac{1}{r} + \frac{n}{pq} < 1$ and $0 < \frac{2}{r} + \frac{n}{q} < 1$	Open Problem
$\text{BMO} \supset L^{\infty,\infty} \simeq L^{\infty}$	Open Problem

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Motivation

One Million Dollar Question:

What should we expect from Nonlinear Scenery ($p \neq 2$)?

We will provide an affirmative answer in the two last sceneries:

$\mathbf{f \in L^{q,r}(Q_1^-)}$	**Sharp Regularity**
$\frac{1}{r} + \frac{n}{pq} < 1$ and $1 < \frac{2}{r} + \frac{n}{q}$	$C_{loc}^{\alpha, \frac{\alpha}{\theta}}$
(CC) $0 < \frac{1}{r} + \frac{n}{pq} < 1$ and $0 < \frac{2}{r} + \frac{n}{q} < 1$	$C_{\mathrm{loc}}^{1+\min\left\{\frac{1-\left(\frac{n}{q}+\frac{2}{r}\right)}{p\left[1-\left(\frac{n}{pq}+\frac{1}{r}\right)\right]-\left[1-\left(\frac{n}{q}+\frac{2}{r}\right)\right]},\, \alpha_{\mathrm{Hom}}^{-}\right\}}$
$\mathrm{BMO} \supset L^{\infty,\infty} \simeq L^{\infty}$	$C_{\mathrm{loc}}^{1+\min\left\{\frac{1}{p-1},\, \alpha_{\mathrm{Hom}}^{-}\right\}}$

Another question:

Are there significant changes between the Teixeira-Urbano's case and the other ones?[a]

[a] Cambia, Todo cambia...Mercedes Sosa. Todo cambia, Live in Europe, 1989.

Motivation

Our Impetus

Therefore, we will focus our attention in establishing sharp (geometric) $C^{1+\alpha}$ regularity estimates for weak solution to (1.1) inside certain critical sets, by using a systematic and modern approach (cf. [1], [2], [3] and [5]).

Motivation

Our Impetus

Therefore, we will focus our attention in establishing sharp (geometric) $C^{1+\alpha}$ regularity estimates for weak solution to (1.1) inside certain critical sets, by using a systematic and modern approach (cf. [1], [2], [3] and [5]).

It is worth highlight that such an estimates play a fundamental role in proving[a]:

1. Blow-up results and Liouville type results;
2. Weak geometric properties (in certain free boundary problems);
3. Hausdorff measure estimates (in certain free boundary problems);

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J.V. da Silva & P. Ochoa, *Fully nonlinear parabolic dead core problems*. To appear in **Pacific J. Math.** 2018.

J.V. da Silva, P. Ochoa & A. Silva, *Regularity for degenerate evolution equations with strong absorption*. **J. Differential Equations** 264 (2018), no. 12, 7270-7293.

Main Theorem

Theorem (Amaral, da S., Ricarte & Teymurazyan, Israel J. Math. 18)

Let $K \subset\subset Q_1^-$, u be a bounded weak solution of (1.1) in Q_1^- and suppose that (CC) are in force. Then u is $C^{1+\alpha}$ (in the parabolic sense), i.e., there exists a (universal) constant $M > 0$ such that

$$[u]^*_{C^{1+\alpha}(K)} \leq M.\left[\|u\|_{L^\infty(Q_1^-)} + \|f\|_{L^{q,r}(Q_1^-)}\right],$$

where

$$[u]^*_{C^{1+\alpha}(K)} := \sup_{0<\rho\leq\rho_0}\left(\inf_{(x_0,t_0)\in\mathcal{C}^\alpha_\rho(Q_1^-)} \frac{\|u - \mathfrak{l}_{(x_0,t_0)}(u)\|_{L^\infty(\hat{Q}^-_\rho(x_0,t_0)\cap K)}}{\rho^{1+\alpha}}\right)$$

and

$$\mathfrak{l}_{(x_0,t_0)}(u)(x) := u(x_0,t_0) + \nabla u(x_0,t_0)\cdot(x - x_0).$$

Chapter 1: Approximation result

A key step in accessing the tangential path toward the regularity theory available for "frozen" coefficient, homogeneous $p-$caloric functions is the following result.

Lemma ($p-$caloric Approximation Lemma)

If u is a weak solution of (1.1) in Q_1^- with $\|u\|_{L^\infty(Q_1^-)} \leq 1$, then $\forall \varepsilon > 0$ there exists $\delta = \delta(p, n, \varepsilon) > 0$ such that whenever $\|f\|_{L^{q,r}(Q_1^-)} \leq \delta$ there exists a p-caloric function $\phi : Q_{\frac{1}{2}}^- \to \mathbb{R}$ such that

$$\max\left\{ \|u-\phi\|_{L^\infty\left(Q_{\frac{1}{2}}^-\right)}, \|\nabla(u-\phi)\|_{L^\infty\left(Q_{\frac{1}{2}}^-\right)} \right\} < \varepsilon. \tag{2.1}$$

Chapter 1: Approximation result

Remark (Normalization and "flatness regime")

Assumptions in the Lemma 3 are not restrictive. Indeed, fixed $\delta > 0$ and $s > 0$, there exists positive constant $\mu = \mu(\delta, s, \|u\|_{L^\infty}, \|f\|_{L^{q,r}})$ such that the function

$$v(x,t) := \mu^s u(\mu^s x, \mu^\tau t),$$

fall into in the conditions of Lemma 2.1, where $\tau := 2s(p-1) > 0$,

$$0 < \mu < \min\left\{1, \frac{1}{\sqrt[s]{\|u\|_{L^\infty(Q_1^-)}}}, \sqrt[\kappa]{\frac{\delta}{\|f\|_{L^{q,r}(Q_1^-)}}}\right\}$$

and

$$\kappa = s\left[(p-1)\left(1-\frac{1}{r}\right)+\frac{1}{r}\right]+sp\left[1-\left(\frac{n}{pq}+\frac{1}{r}\right)\right].$$

Chapter 2: Metric (a priori estimate) Vs Geometry of parabolic cylinder

Lemma (Pseudo first step of induction)

Let u be a weak solution of (1.1) in Q_1^- with $\|u\|_{L^\infty(Q_1^-)} \leq 1$. There exist $\delta > 0$ and $\rho \in \left(0, \frac{1}{2}\right)$ such that if $\|f\|_{L^{q,r}(Q_1^-)} \leq \delta$, then

$$\sup_{\hat{Q}_\rho(x_0,t_0)} \left| u(x,t) - \mathfrak{l}_{(x_0,t_0)}(u)(x) \right| \leq \rho^{1+\alpha}.$$

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Chapter 2: Metric (a priori estimate) Vs Geometry of parabolic cylinder

Idea of proof.

For $Q_\rho^-(x_0,t_0) = B_\rho(x_0) \times (t_0 - \rho^\theta, t_0]$ with $\theta > 0$ (intrinsic scaling factor):

$$\begin{aligned}
\left\|u - \mathfrak{l}_{(x_0,t_0)}(u)\right\|_{L^\infty(Q_\rho(x_0,t_0))} &\leq \left\|\phi - \mathfrak{l}_{(x_0,t_0)}(\phi)\right\|_{L^\infty(Q_\rho(x_0,t_0))} + |(u-\phi)(x_0,t_0)| \\
&+ \|u-\phi\|_{L^\infty(Q_\rho(x_0,t_0))} + |\nabla(u-\phi)(x_0,t_0)| \\
&\leq C \sup_{Q_\rho(x_0,t_0)} \left(|x-x_0| + \sqrt{|t-t_0|}\right)^{1+\alpha_{\text{Hom}}} + 3\varepsilon \\
&\leq C\rho^{(1+\alpha_{\text{Hom}})\min\{1,\frac{\theta}{2}\}} + 3\varepsilon \\
&\leq C\rho^{(1+\alpha_{\text{Hom}})} + 3\varepsilon \ \text{(expected estimate)}
\end{aligned}$$

□

Chapter 2: Metric (a priori estimate) Vs Geometry of parabolic cylinder

Idea of proof.

Notice that for $p > 2$

$$1 < 2 + (2-p)\hat{\alpha} \leq \theta(\alpha, p, \rho, \|\nabla u\|) \leq 2.$$

In this point, we define the intrinsic correction factor for our (corrected) parabolic cylinders:

$$\sigma := \frac{2}{2 + (2-p)\hat{\alpha}} \in [1, 2), \text{ where } \hat{\alpha} := \frac{1 - \left(\frac{n}{q} + \frac{2}{r}\right)}{p\left[1 - \left(\frac{n}{pq} + \frac{1}{r}\right)\right] - \left[1 - \left(\frac{n}{q} + \frac{2}{r}\right)\right]}.$$

Such a definition assures that $\theta\sigma \geq 2$, which allow us put the parabolic cylinder in the correct framework. $\square$

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Chapter 2: Metric (a priori estimate) Vs Geometry of parabolic cylinder

Idea of proof.

Coming back to our estimate (now with the corrected cylinder)

$$\hat{Q}^-_{\rho^l}(x_0,t_0) := B_{\rho^l}(x_0) \times \left(t_0 - \rho^{\theta(\sigma+l-1)}, t_0\right] \subset Q^-_{\rho^l}(x_0,t_0)$$

we can conclude:

$$\begin{aligned} \left\| u - \mathfrak{l}_{(x_0,t_0)}(u) \right\|_{L^\infty(\hat{Q}_\rho(x_0,t_0))} &\leq C\rho^{(1+\alpha_{\text{Hom}})\min\left\{1, \frac{\theta\sigma}{2}\right\}} + 3\varepsilon \\ &\leq C\rho^{(1+\alpha_{\text{Hom}})} + 3\varepsilon \\ &\leq \rho^{1+\alpha} \end{aligned}$$

provided

$$\rho \in \left(0, \min\left\{\frac{1}{2}, \left(\frac{1}{2C}\right)^{\frac{1}{\alpha_{\text{Hom}}-\alpha}}\right\}\right) \quad \text{and} \quad \varepsilon \in \left(0, \frac{1}{6}\rho^{1+\alpha}\right).$$

.

□

Chapter 3: The gap in the standard induction process

Different from $C^{1+\zeta}$ regularity estimates proved in the linear setting, we should point out that the former lemma is not enough to proceed with an iterative scheme, because a priori we do not know the equation which would be satisfied by

$$v_k(x,t) := \frac{u(\rho^k x + x_0, \rho^{k\theta} t + t_0) - \mathfrak{l}_k(\rho^k x + x_0)}{\rho^{k(1+\alpha)}},$$

where $\{\mathfrak{l}_k\}_{k\in\mathbb{N}}$ is sequence of affine functions.

Chapter 4: Iteration: A new oscillation mechanism

Corollary (First (real) step of induction)

Suppose that the assumptions of previous Lemma are in force. Then,

$$\sup_{\hat{Q}_\rho^-(x_0,t_0)} |u(x,t) - u(x_0,t_0)| \leq \rho^{1+\alpha} + \rho|\nabla u(x_0,t_0)|.$$

In order to obtain a precise control on the influence of magnitude of the gradient of u, we iterate solutions (using the previous Corollary) in corrected ρ-adic cylinders.

Lemma (Iterative process)

Under the assumptions of previous Corollary one has

$$\sup_{\hat{Q}_{\rho^k}^-(x_0,t_0)} |u(x,t) - u(x_0,t_0)| \leq \rho^{k(1+\alpha)} + |\nabla u(x_0,t_0)| \sum_{j=0}^{k-1} \rho^{k+j\alpha}. \tag{2.2}$$

Chapter 4: Iteration: A new oscillation mechanism

Our next result provides the geometric regularity estimate inside critical zone. We define the critical zone as follows:

$$\mathcal{C}^{\alpha}_{\rho}(Q_1^-) := \left\{(x,t) \in Q_1^- ; \, |\nabla u(x,t)| \leq \rho^{\alpha}\right\}.$$

Theorem

Suppose that the assumptions of previous Lemma are in force. Then, there exists a universal constant $M > 1$ such that

$$\sup_{\hat{Q}^-_{\rho_0}(x_0,t_0)} |u(x,t) - u(x_0,t_0)| \leq M{\rho_0}^{1+\alpha}\left(1 + |\nabla u(x_0,t_0)|{\rho_0}^{-\alpha}\right), \; \forall \rho_0 \in (0,\rho).$$

Proof of Main Theorem

Proof of the Theorem.

WLOG, we may assume that $K = Q^-_{\frac{1}{2}}$ and $(x_0, t_0) = (0, 0)$. Using previous Theorem (re-scaled according to Remark, if needed), we estimate

$$\begin{aligned}
\sup_{\hat{Q}^-_{\rho_0}} \frac{\left|u(x,t) - \mathfrak{l}_{(0,0)} u(x)\right|}{\rho^{1+\alpha}} &\leq \sup_{\hat{Q}^-_{\rho_0}} \frac{|u(x,t) - u(0,0)|}{\rho_0^{1+\alpha}} + \frac{|\nabla u(0,0)|\rho_0}{\rho_0^{1+\alpha}} \\
&\leq M\left(1 + |\nabla u(0,0)|\rho_0^{-\alpha}\right) + 1 \\
&\leq 3M
\end{aligned}$$

□

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Final Chapter: The Journey Continues...

Coming back to the open issues:

$\mathbf{f \in L^{q,r}(Q_1^-)}$	**Sharp Regularity**
$\frac{1}{r} + \frac{n}{pq} = 1$ and $1 < \frac{2}{r} + \frac{n}{q}$	Open Problem
$\frac{1}{r} + \frac{n}{pq} < 1$ and $1 < \frac{2}{r} + \frac{n}{q}$	$C_{loc}^{\alpha, \frac{\alpha}{\theta}}$
$\frac{1}{r} + \frac{n}{pq} < 1$ and $1 = \frac{2}{r} + \frac{n}{q}$	Open Problem
(CC) $0 < \frac{1}{r} + \frac{n}{pq} < 1$ and $0 < \frac{2}{r} + \frac{n}{q} < 1$	$C_{\text{loc}}^{1+\min\left\{\frac{1-\left(\frac{n}{q}+\frac{2}{r}\right)}{p\left[1-\left(\frac{n}{pq}+\frac{1}{r}\right)\right]-\left[1-\left(\frac{n}{q}+\frac{2}{r}\right)\right]}, \alpha_{\text{Hom}}^{-}\right\}}$
$\text{BMO} \supset L^{\infty,\infty} \simeq L^{\infty}$	$C_{\text{loc}}^{1+\min\left\{\frac{1}{p-1}, \alpha_{\text{Hom}}^{-}\right\}}$

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Expected regularity estimates

$\mathbf{f \in L^{q,r}(Q_1^-)}$	**Sharp Regularity**
$\frac{1}{r} + \frac{n}{pq} = 1$ and $1 < \frac{2}{r} + \frac{n}{q}$	BMO_{loc}
$\frac{1}{r} + \frac{n}{pq} < 1$ and $1 < \frac{2}{r} + \frac{n}{q}$	$C_{loc}^{\alpha, \frac{\alpha}{\theta}}$
$\frac{1}{r} + \frac{n}{pq} < 1$ and $1 = \frac{2}{r} + \frac{n}{q}$	Log-Lipschitz type estimate
(CC) $0 < \frac{1}{r} + \frac{n}{pq} < 1$ and $0 < \frac{2}{r} + \frac{n}{q} < 1$	$C_{\text{loc}}^{1+\min\left\{\frac{1-\left(\frac{n}{q}+\frac{2}{r}\right)}{p\left[1-\left(\frac{n}{pq}+\frac{1}{r}\right)\right]-\left[1-\left(\frac{n}{q}+\frac{2}{r}\right)\right]},\, \alpha_{\text{Hom}}^-\right\}}$
$\text{BMO} \supset L^{\infty,\infty} \simeq L^{\infty}$	$C_{\text{loc}}^{1+\min\left\{\frac{1}{p-1},\, \alpha_{\text{Hom}}^-\right\}}$

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