# POINCARÉ, SOBOLEV AND RUBIO DE FRANCIA

In this talk I will present some recents results on weighted Poincaré and Poincaré-Sobolev type inequalities with an explicit analysis on the dependence on the $A_p$ constants of the involved weights. We obtain inequalities of the form

$$\left(\frac{1}{w(Q)}\int_Q |f-f_Q|^q w\right)^{\frac{1}{q}} \leq C_w \ell(Q)\left(\frac{1}{w(Q)}\int_Q |\nabla f|^p w\right)^{\frac{1}{p}},$$

with different quantitative estimates for both the exponent $q$ and the constant $C_w$. We will derive those estimates together with a large variety of related results as a consequence of a general selfimproving property shared by functions satisfying the inequality

$$\fint_Q |f-f_Q| d\mu \leq a(Q),$$

for all cubes $Q \subset \mathbb{R}^n$ and where $a$ is some functional that obeys a specific discrete geometrical summability condition. For the endpoint case of $A_1$ weights we reach the classical critical Sobolev exponent $p^* = \frac{pn}{n-p}$ which is the largest possible and provide different type of quantitative estimates for $C_w$.

We will also discuss an interesting application of a variation of the famous extrapolation technique of Rubio de Francia to a problem related to the Keith-Zhong theorem on the open ended condition for Poincaré inequalities.

This is a joint work with Carlos Pérez Moreno from BCAM, Bilbao, Spain.