Approximation by group invariant subspaces

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Abstract

We consider the problem of best approximation of a dataset in $L^2(\mathbb{R}^n)$ by closed linear subspaces that are invariant under translations by a lattice of $\mathbb{R}^n$ and under an additional group of automorphisms of $\mathbb{R}^n$ that preserves the lattice of translations. The whole set of symmetries defines a group that in general is not abelian, and has the structure of a semidirect product. A notable class of examples is provided by crystallographic groups. We address this problem in terms of range functions possessing an additional covariance property, and obtain the generators of the invariant approximation space by adapting an infinite dimensional version of Eckhart-Young Theorem. A motivation for this work is provided by datasets which possess non abelian symmetries based on a translational structure, such as planar translations and rotations for natural images.

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