Partial Characterizations of Circular-Arc Graphs

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ABSTRACT

A circular-arc graph is the intersection graph of a family of arcs on a circle. A characterization by forbidden induced subgraphs for this class of graphs is not known, and in this work we present a partial result in this direction. We characterize circular-arc graphs by a list of minimal forbidden induced subgraphs when the graph belongs to any of the following classes: P_4-free graphs, paw-free graphs, claw-free chordal graphs and diamond-free graphs. © *X John Wiley & Sons, Inc.*

Keywords: circular-arc graphs, claw-free chordal graphs, cographs, diamond-free graphs, paw-free graphs.

1. INTRODUCTION

A graph G is a *circular-arc* (CA) graph if it is the intersection graph of a set $\mathcal{S}$ of arcs on a circle, i.e., if there exists a one-to-one correspondence between the vertices of G and the arcs of $\mathcal{S}$ such that two vertices of G are adjacent if and only if the corresponding arcs in $\mathcal{S}$ intersect. Such a family of arcs is called a *circular-arc model* (CA model) of G. CA graphs can be recognized in linear time [13]. A graph is an *interval* graph if it is the intersection graph of a set of intervals on the real line. Equivalently, a graph is an interval graph if it admits a CA model such that the set of arcs does not cover the circle. Interval graphs have been characterized by minimal forbidden induced subgraphs [11]. A graph G is a *proper circular-arc* (PCA) graph if it admits a CA model in which no arc is contained in another arc. Tucker gave a characterization of PCA graphs by minimal forbidden induced subgraphs [18]. Furthermore, this subclass can be recognized in linear time [7]. A graph G is a *unit circular-arc* (UCA) graph if it admits a CA model in which all the arcs have the same length. Tucker gave a characterization by minimal forbidden induced subgraphs for this class [18]. Recently, linear and quadratic-time recognition algorithms for this class have been shown [12, 8]. Finally, the class of CA graphs that are complements of bipartite graphs was characterized by minimal forbidden induced subgraphs [17].

Nevertheless, the problem of characterizing the whole class of CA graphs by forbidden induced subgraphs remains open. In this work we present some steps in this direction by providing characterizations of CA graphs by minimal forbidden induced subgraphs when the graph belongs to any of four different classes: P_4-free graphs, paw-free graphs, claw-free chordal graphs and diamond-free graphs. All of these classes were studied along the way towards the proof of the Strong Perfect Graph Theorem [4, 14, 15, 16, 19].

2. DEFINITIONS

Let G be a finite, simple, loopless, undirected graph, with vertex set $V(G)$ and edge set $E(G)$. The graph G will be called *empty* if $V(G) = \emptyset$ and *trivial* if $|V(G)| = 1$. Denote by $N(v)$ the set of neighbours of $v \in V(G)$. $G[W]$ denotes the subgraph of G induced by W. For any $W \subseteq V(G)$, denote by $\overline{G}$ the complement of G. If H is an induced subgraph of G and v a vertex of G, we denote by $N_H(v)$ the set $N(v) \cap V(H)$ and by $G - H$ the graph $G[V(G) - V(H)]$.

Let $A, B \subseteq V(G)$. We say that A *is complete to* B if every vertex of A is adjacent to every vertex of B; and A *is anticomplete to* B if A is complete to B in $\overline{G}$. A *stable set* is a subset of pairwise non-adjacent vertices. A graph G is *bipartite* if $V(G)$ can be partitioned into two stable sets V_1, V_2; G is *complete bipartite* if V_1 is complete to V_2. Denote by $K_{r,s}$ the complete bipartite graph with $|V_1| = r$ and $|V_2| = s$. A *claw* is the complete bipartite graph $K_{1,3}$.

Denote by K_r ($r \geq 0$) the complete graph on r vertices; K_3 will be also called a *triangle*. A *clique* is a subset of vertices inducing a complete subgraph. A *paw* is the graph obtained from a triangle T by adding a vertex adjacent to exactly one vertex of T. A *diamond* is the graph obtained from a complete K_4 by removing exactly one edge.

Let $P = v_1 \dots v_k$ be a path. Vertices v_1 and v_k will be called the *endpoints* of P, while $V(P) - \{v_1, v_k\}$ will be called the *interior points* of P. Denote by $|P|$ the number of vertices of P. An edge joining two non-consecutive vertices of a path or a cycle in a graph will be called a *chord*. An *induced path* is a chordless path in a graph. Likewise, an *induced cycle* is a chordless cycle in a graph. The graph P_4 is an induced path on 4 vertices. A *hole* is an induced cycle of length at least 4. A graph is *chordal* if it does not contain any hole.

Let G and H be two graphs; we say that G is H-free if G does not contain an induced subgraph isomorphic to H. If $\mathcal{H}$ is a family of graphs, we say that G is $\mathcal{H}$-free if G is H-free for every $H \in \mathcal{H}$.

Denote by G^* the graph obtained from G by adding an isolated vertex. If t is a non-negative integer, then tG will denote the disjoint union of t copies of G. A graph G is a *multiple* of another graph H if G can be obtained from H by replacing each vertex x of H by a non-empty complete graph M_x and adding all possible edges between M_x and M_y if and only if x and y are adjacent in H.

A *universal vertex* is a vertex adjacent to every other vertex of the graph. Let G and H be graphs. G is an *augmented* H if G is isomorphic to H or if G can be obtained from H by repeatedly adding a universal vertex. G is a *bloomed* H if there exists a subset $W \subseteq V(G)$ such that $G[W]$ is isomorphic to H and $V(G) - W$ is either empty or it induces in G a disjoint union of non-empty complete graphs $B_1, B_2, \dots, B_j$ for some $j \geq 1$, where each B_i is complete to one vertex of $G[W]$, but anticomplete to any other vertex of $G[W]$. If each vertex in W is complete to at least one of $B_1, B_2, \dots, B_j$, we say that G is a *fully bloomed* H. The complete graphs $B_1, \dots, B_j$ will be referred as the *blooms*. A bloom is *trivial* if it is composed of only one vertex.

Given two graphs G and H such that $V(G) \cap V(H) = \emptyset$, the *disjoint union* $G \cup H$ is the graph with vertex set $V(G) \cup V(H)$ and edge set $E(G) \cup E(H)$. The *join* of G and H is obtained from $G \cup H$ by adding all the edges between $V(G)$ and $V(H)$.

A graph G is *anticonnected* if $\overline{G}$ is connected; an *anticomponent* of G is the subgraph of G induced by the vertices of a component of $\overline{G}$.

3. PRELIMINARY RESULTS

Special graphs needed throughout this work are depicted in Figures 1 and 2. We use *net* and *tent* as abbreviations for 2-net and 3-tent, respectively.

Bang-Jensen and Hell proved the following result.

Theorem 1. [1] Let G be a connected graph containing no induced claw, net, C_4, or C_5. If G contains a tent as induced subgraph, then G is a multiple of a tent.

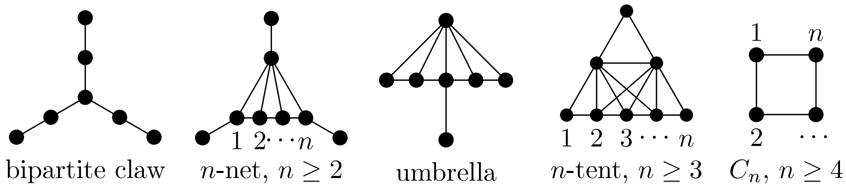

FIGURE 1. Minimal forbidden induced subgraphs for the class of interval graphs.

Theorem 1 allows to provide the following description of all the minimal non-PCA graphs within the class of connected chordal graphs.

Theorem 2. [1] Let G be a connected chordal graph. Then, G is PCA if and only if it contains no induced claw or net.

Lekkerkerker and Boland determined all the minimal forbidden induced subgraphs for the class of interval graphs.

Theorem 3. [11] The minimal forbidden induced subgraphs for the class of interval graphs are: bipartite claw, n-net for every $n \geq 2$, umbrella, n-tent for every $n \geq 3$, and C_n for every $n \geq 4$ (cf. Figure 1).

This characterization yields some minimal forbidden induced subgraphs for the class of CA graphs. Let H be a minimal forbidden induced subgraph for the class of interval graphs. Note that if H is non-CA, then H is minimally non-CA; i.e., all of its proper induced subgraphs are CA. Otherwise, if H is CA, then H^* is minimally non-CA, and furthermore all non-connected minimally non-CA graphs are obtained this way. Since the umbrella, net, n-tent for all $n \geq 3$, and C_n for all $n \geq 4$ are CA, but the bipartite claw and n-net for all $n \geq 3$ are not, this observation and Theorem 3 lead to the following result.

Corollary 4. [17] The following graphs are minimally non-CA graphs: bipartite claw, net*, n-net for all $n \geq 3$, umbrella*, (n-tent)* for all $n \geq 3$, and C_n^* for every $n \geq 4$. Any other minimally non-CA graph is connected.

We call the graphs listed in Corollary 4 *basic* minimally non-CA graphs. Any other minimally non-CA graph will be called *non-basic*. The following result, which gives a structural property for all non-basic minimally non-CA graphs, can be deduced from Theorem 3 and Corollary 4.

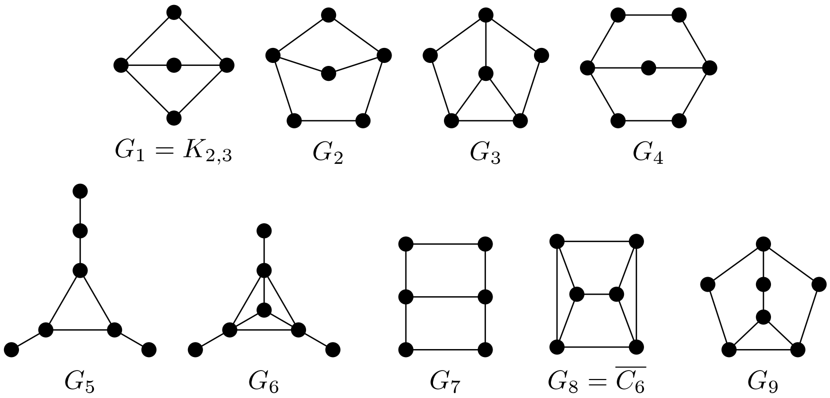

FIGURE 2. Some minimally non-CA graphs.

Corollary 5. If G is a non-basic minimally non-CA graph, then G has an induced subgraph H that is isomorphic to an umbrella, a net, a j-tent for some $j \geq 3$, or C_j for some $j \geq 4$. In addition, each vertex v of $G - H$ is adjacent to at least one vertex of H.

Proof. Since G is non-CA, in particular, G is not an interval graph. By Theorem 3, G has an induced subgraph H isomorphic to a bipartite claw, umbrella, j-net for $j \geq 2$, j-tent for $j \geq 3$, or C_j for some $j \geq 4$. Since G is non-basic minimally non-CA, H is isomorphic to umbrella, net, j-tent for some $j \geq 3$, or C_j for some $j \geq 4$. Moreover, since G is not isomorphic to H^*, every vertex v of $G - H$ is adjacent to at least one vertex of H. ∎

Figure 2 introduces the graphs G_i, for $i \in \{1, 2, \ldots, 9\}$.

Theorem 6. Let G be a minimally non-CA graph. If G is not isomorphic to $K_{2,3}$, G_2, G_3, G_4, or C_j^*, for $j \geq 4$, then for every hole H of G and for each vertex v of $G - H$, either v is complete to H, or $N_H(v)$ induces a non-empty path in H.

Proof. Let G be a minimally non-CA graph, and suppose that G is not isomorphic to $K_{2,3}$, G_2, G_3, G_4, or C_j^* for $j \geq 4$. Suppose, by way of contradiction, that there is a hole H of G and there is a vertex v of $G - H$ such that v is not complete to H and $N_H(v)$ does not induce a path in H. Note that $N_H(v)$ is non-empty because G is minimally non-CA and it is not isomorphic to C_j^* for $j \geq 4$.

So, $H - N_H(v)$ is non-empty and is neither a path nor a hole, hence it is not connected. Let Q_1 and Q_2 be two components of $H - N_H(v)$. Then, there are induced paths P^1 and P^2 on H such that the interior vertices of P^i are Q_i, for $i = 1, 2$. Therefore, the following conditions hold:

(1) each of P^1 and P^2 has at least three vertices,
(2) v is adjacent to none of the interior vertices of P^1 and P^2, and
(3) v is adjacent to the endpoints of P^1 and the endpoints of P^2.

By definition, P^1 and P^2 have no interior vertices in common.

Suppose, by way of contradiction, that P^1 and P^2 have no common endpoints. Let w be an interior vertex of P^1, so w is anticomplete to the hole induced by $\{v\} \cup V(P^2)$ on G. Then, $\{v, w\} \cup V(P^2)$ induces a proper subgraph of G (it is proper since it does not contain the endpoints of P^1) that is not a CA graph, a contradiction.

Suppose next that P^1 and P^2 have exactly one endpoint in common. Suppose, by way of contradiction, that P^1 has at least two interior vertices. Then, there is an interior vertex w of P^1 that is non-adjacent to the common endpoint of P^1 and P^2. Since $\{w\}$ is anticomplete to $\{v\} \cup V(P^2)$, $\{v, w\} \cup V(P^2)$ induces a proper subgraph in G (it is proper because it does not contain the endpoint of P^1 that is not a vertex of P^2) that is non-CA, a contradiction. This contradiction proves that each one of P^1 and P^2 has exactly one interior vertex. Then, $\{v\} \cup V(P^1) \cup V(P^2)$ would induce on G a subgraph isomorphic to either G_3 or G_7, both of which are non-CA graphs. Since G is minimally non-CA, $V(G) = \{v\} \cup V(P^1) \cup V(P^2)$. Since $V(P^1) \cup V(P^2) \subseteq V(H)$, necessarily $V(H) = V(P^1) \cup V(P^2)$. Since H induces a hole in G, G is isomorphic to G_3, a contradiction.

Finally suppose that P^1 and P^2 have exactly two endpoints in common. Suppose, by way of contradiction, that P^1 has more than two interior points. Let w be an interior vertex of P^1 that is adjacent to none of its endpoints. Then, w is anticomplete to $\{v\} \cup V(P^2)$ and thus $\{v, w\} \cup V(P^2)$ induces a proper subgraph on G (it is proper because it does not contain the neighbours of w in H) that is non-CA, a contradiction. This contradiction shows that each one of P^1 and P^2 has at most two interior vertices. Thus, $\{v\} \cup V(P^1) \cup V(P^2)$ induces on G either $K_{2,3}$, G_2 or G_4, which are minimally non-CA graphs. Since G is minimally non-CA, G is isomorphic to one of them, a contradiction. ∎

4. PARTIAL CHARACTERIZATIONS

4.1. Cographs

A *cograph* is a graph with no induced P_4. Cographs were studied in several previous works as, e.g., [5, 6, 16]. Seinsche proved the following well-known fact about cographs.

Theorem 7. [16] If G is a cograph, then G is either not connected or not anticonnected.

Define *semicircular graphs* to be the intersection graphs of open semicircles on a circle. By definition, semicircular graphs are UCA graphs.

Theorem 8. Let G be a graph. The following conditions are equivalent:

(1) G is $\{P_4, 3K_1\}$-free.
(2) G is an augmented multiple of $\overline{tK_2}$ for some non-negative integer t.
(3) G is a semicircular graph.

Proof. (1) $\Rightarrow$ (2) Assume that G is a $\{P_4, 3K_1\}$-free graph. If G has less than two vertices, then G is a complete (note that $\overline{tK_2}$ with $t = 0$ is an empty graph). So, we can assume that G has at least two vertices. Since G is P_4-free, by Theorem 7, G is either not connected or not anticonnected.

Since G is $3K_1$-free, if G is not connected, then G has exactly two components. Moreover, both components are complete graphs. Thus, G is a multiple of $\overline{K_2}$. Suppose now that G is non-anticonnected, and let H be an anticomponent of G. Since H is $\{P_4, 3K_1\}$-free and anticonnected, H is either trivial or non-connected and, in the second case, by the arguments above H induces on G a multiple of $\overline{K_2}$. Let s be the number of anticomponents of G that are trivial and t be the number of anticomponents of G that induce on G a multiple of $\overline{K_2}$. Since G is the join of its anticomponents, G is the join of a multiple of $\overline{tK_2}$ and a complete K_s for some non-negative integers t and s. Equivalently, G is an augmented multiple of $\overline{tK_2}$ for some non-negative integer t.

(2) $\Rightarrow$ (3) Assume that G is an augmented multiple of $\overline{tK_2}$ for some non-negative t. In particular, G is a multiple of $\overline{tK_2 \cup sK_1}$ for some non-negative t and some $s = 0$ or 1. In order to prove that G is a semicircular graph, it will suffice to prove that $\overline{tK_2 \cup sK_1}$ is a semicircular graph. Fix a circle $\mathcal{C}$. Let $\{p_1, p_1'\}, \ldots, \{p_t, p_t'\}, \{q_1, q_1'\}, \ldots, \{q_s, q_s'\}$ be $t + s$ pairwise distinct pairs of antipodal points of $\mathcal{C}$. For $i = 1, \ldots, t$, let S_i^1 and S_i^2 be the two disjoint open semicircles on $\mathcal{C}$ whose endpoints are p_i and p_i'. For $j = 1, \ldots, s$ let T_j be an open semicircle on $\mathcal{C}$ whose endpoints are q_j and q_j'. Then $S_1^1, S_1^2, \ldots, S_t^1, S_t^2, T_1, \ldots, T_s$ is a semicircular model for $\overline{tK_2 \cup sK_1}$.

(3) $\Rightarrow$ (1) We now prove that semicircular graphs are $\{P_4, 3K_1\}$-free graphs. It is clear that $3K_1$ is not a semicircular graph because there is not enough space on a circle for three pairwise disjoint semicircles. We now show that P_4 is not a semicircular graph. Assume, by way of contradiction, that there is a semicircular graph model for P_4. Let $V(P_4) = \{v_1, v_2, v_3, v_4\}$, where v_i is adjacent to v_{i+1} for $i = 1, 2, 3$ and let $\mathcal{S} = \{S_1, S_2, S_3, S_4\}$ be a semicircular model for P_4, where the semicircle S_i corresponds to the vertex v_i. Since v_1 and v_3 are non-adjacent, S_1 and S_3 are disjoint and have the same endpoints. Since v_1 and v_4 are also non-adjacent, the same holds for S_1 and S_4, hence $S_3 = S_4$. This contradicts the fact that $S_2 \cap S_3$ is non-empty but $S_2 \cap S_4$ is empty. This contradiction shows that P_4 is not a semicircular graph. Since the class of semicircular graphs is hereditary, a semicircular graph is $\{3K_1, P_4\}$-free. ∎

Theorem 9. Let G be a cograph that contains an induced C_4, and such that all its proper induced subgraphs are CA graphs. Then, exactly one of the following conditions holds:

(1) G is isomorphic to $K_{2,3}$ or C_4^*.
(2) G is an augmented multiple of $\overline{tK_2}$, for some integer $t \geq 2$.

Proof. Clearly, $K_{2,3}$ and C_4^* are not augmented multiples of $\overline{tK_2}$, for any integer $t \geq 2$.

Assume that G is isomorphic to neither C_4^* nor $K_{2,3}$. Since all proper induced subgraphs of G are CA graphs, C_4^* and $K_{2,3}$ are not proper induced subgraphs of G.

We must prove that G is an augmented multiple of $\overline{tK_2}$, for some integer $t \geq 2$. Let H be the induced subgraph of G that is isomorphic to C_4. Since $C_4 = \overline{2K_2}$, we may suppose that there is a vertex v in $G - H$. Since C_4^* is not an induced subgraph of G, v is adjacent to at least one vertex of H. Since G is $\{P_4, K_{2,3}\}$-free, either v is adjacent to three vertices of H or v is complete to H. In case that v is adjacent to three vertices of H we will denote by $C(v)$ the interior vertex of the path induced by $N_H(v)$ in H.

Suppose there exists a vertex w of $G - H$, $w \neq v$, that is non-adjacent to v. If v were adjacent to three vertices of H and w were complete to H, then the subgraph induced by $\{v, w\} \cup V(H)$ in G would contain an induced P_4, a contradiction. Thus, v and w are both adjacent to three vertices of H or they are both complete to H. Next assume that v and w are both adjacent to three vertices of G. If $C(v) = C(w)$, then $\{v, w\} \cup (V(H) - \{C(v)\})$ would induce in G a graph isomorphic to $K_{2,3}$. If $C(v)$ and $C(w)$ were adjacent, then $\{v, w\} \cup V(H)$ would contain an induced P_4. We conclude that if v and w are both adjacent to three vertices of H, then $C(v)$ and $C(w)$ must be distinct and non-adjacent vertices of H.

We now prove that G does not contain $3K_1$ as induced subgraph. Assume, by way of contradiction, that there is an induced subgraph S of G isomorphic to $3K_1$. Clearly H and S have at most two vertices in common. If H and S had two vertices in common, then the remaining vertex of S would be a vertex of $G - H$ adjacent to at most two vertices of H, a contradiction. If H and S had exactly one vertex in common, then the other two vertices of S would be adjacent to the same three vertices of H. As we noticed above, this leads to a contradiction. We conclude that H and S must have no vertices in common. Let $\{v_1, v_2, v_3\} = V(S)$. Since the vertices of S are vertices of $G - H$ and pairwise non-adjacent, all of them are adjacent to three vertices of H or all of them are complete to H. If all of them were adjacent to three vertices of H, then $C(v_1)$, $C(v_2)$, $C(v_3)$ would be pairwise distinct and non-adjacent vertices of H, a contradiction. If all of them were complete to H, then $H \cup \{v_1, v_2, v_3\}$ induces in G a graph which contains an induced $K_{2,3}$, a contradiction. We conclude that G is $3K_1$-free. Since G is also P_4-free, by Theorem 8, G is an augmented multiple of $\overline{tK_2}$. Finally, since G contains C_4 as an induced subgraph, $t \geq 2$. ∎

We can now prove the main result of this section.

Corollary 10. Let G be a cograph. Then, G is a CA graph if and only if G contains no induced $K_{2,3}$ or C_4^*.

Proof. Let H be a cograph. Suppose, by way of contradiction, that H is a minimally non-CA graph but H is not isomorphic to $K_{2,3}$ or C_4^*. Since H is not an interval graph and it is P_4-free, by Theorem 3, H contains an induced C_4. By Theorem 9, H is an augmented multiple of $\overline{tK_2}$, for some $t \geq 2$. Thus, by Theorem 8, H is a CA graph, a contradiction. ∎

4.2. Paw-free graphs

A *paw-free* graph is a graph with no induced paw. Paw-free graphs were studied in [14].

Theorem 11. Let G be a paw-free graph containing an induced C_4 and such that all its proper induced subgraphs are CA graphs. Then, at least one of the following conditions holds:

(1) G is isomorphic to $K_{2,3}$, G_2, G_7, or C_4^*.
(2) G is a bloomed C_4 with trivial blooms.
(3) G is an augmented multiple of $\overline{tK_2}$ for some $t \geq 2$.

Proof. Assume that G is not isomorphic to $K_{2,3}$, G_2, G_7, or C_4^*. Since all proper induced subgraphs of G are CA, G does not contain any of these graphs as induced subgraphs.

Let H be an induced subgraph of G isomorphic to C_4. If $G = H$, then the theorem holds. Otherwise, let v be any vertex of $G - H$. Since G is C_4^*-free, v is adjacent to at least one vertex of H. Since G is paw-free, v cannot be adjacent to either exactly three vertices of H or exactly two adjacent vertices of H. Since G is $K_{2,3}$-free, v cannot be adjacent to exactly two non-adjacent vertices of H. We conclude that each vertex v of $G - H$ is either adjacent to exactly one vertex of H or complete to H.

Suppose that there are two vertices w, w' in $G - H$ such that w is complete to H and w' is adjacent to exactly one vertex x of H. If w and w' are non-adjacent, then w, w', x and any neighbour of x in H induce a paw in G; if w and w' are adjacent, then w, w', x and the non-neighbour of x in H induce a paw in G. Since G is paw-free, either all vertices of $G - H$ are complete to H, or each vertex of $G - H$ is adjacent to exactly one vertex of H (not necessarily all of them to the same vertex).

Assume first that each vertex of $G - H$ is adjacent to exactly one vertex of H. Let us prove that $G - H$ is a stable set. Assume, by way of contradiction, that there are two adjacent vertices v and w in $G - H$. Since G is paw-free, v and w cannot be adjacent to the same vertex. Since G contains no induced G_7, v and w must be adjacent to non-adjacent vertices of H. Similarly, since G contains no induced G_2, v and w cannot be adjacent to non-adjacent vertices of H, a contradiction. Thus, $G - H$ is a stable set. Since each vertex of $G - H$ is adjacent to exactly one vertex of H, G is a bloomed C_4 with trivial blooms.

Assume now that all vertices of $G - H$ are complete to H. If $G - H$ contains three pairwise non-adjacent vertices, then these vertices and two non-adjacent vertices of H induce $K_{2,3}$, a contradiction. If $G - H$ contains P_4, then three non-consecutive vertices of P_4 and any vertex of H induce a paw, a contradiction. Thus, $G - H$ is $\{3K_1, P_4\}$-free. Since H is complete to $G - H$, every induced subgraph of G with at least one vertex in H and at least one vertex in $G - H$ is non-anticonnected. Since P_4 and $3K_1$ are anticonnected, if G contains an induced subgraph isomorphic to either $3K_1$ or P_4, then it must be entirely contained in either H or $G - H$. As observed above, this situation is not possible, hence G is $\{3K_1, P_4\}$-free. By Theorem 8, G is an augmented multiple of $\overline{tK_2}$ for some non-negative t. Finally, since G contains an induced C_4, $t \geq 2$. ∎

Theorem 12. Let $k \geq 5$. Let G be a paw-free graph such that all its proper induced subgraphs are CA graphs. If G contains an induced subgraph H isomorphic to C_k, then exactly one of the following conditions holds:

(1) G is isomorphic to G_2, G_4, or C_k^*.
(2) G is a bloomed C_k with trivial blooms.

Proof. Assume that G is not isomorphic to G_2, G_4, or C_k^*. Since all proper induced subgraphs of G are CA, G does not contain any of these graphs as induced subgraph. Moreover, G contains no induced C_j^*, for any $j \geq 4$. G is paw-free, so it is not isomorphic to G_3; G contains an induced cycle of length at least five, so it is not isomorphic to $K_{2,3}$. If $G = H$, then the theorem holds. Otherwise, by Theorem 6, if v is a vertex of $G - H$, then either v is complete to H or $N_H(v)$ induces a non-empty path on H. But, since H is isomorphic to C_k, $k \geq 5$, and G is paw-free, every vertex of $G - H$ must be adjacent to exactly one vertex of H.

We will show now that $G - H$ is a stable set of G. Let v and w be two vertices of $G-H$. Suppose, by way of contradiction, that v and w are adjacent. Since G is paw-free, v and w cannot be adjacent to the same vertex of H. If v and w were adjacent to two adjacent vertices of H, then G would properly contain an induced C_4^*. We can assume now that v and w are adjacent to non-adjacent vertices of H. Let P^1 and P^2 be the two distinct paths joining the neighbours of v and w within H. By hypothesis, each of P^1 and P^2 has at least three vertices, and at least one of them has four vertices, since H has at least five vertices. Since G contains no induced $C_j^*, j \geq 4$, each of P^1 and P^2 has at most four vertices. If P^1 and P^2 have three and four vertices respectively, then $\{v, w\} \cup V(H)$ would induce in G the graph G_4, a contradiction. Finally, if each of P^1 and P^2 has four vertices, then $\{v, w\} \cup V(H) - N_H(v)$ induces properly on G a bipartite claw, a contradiction. We conclude that $G-H$ is a stable set of G, and since each vertex of $G-H$ is adjacent to exactly one vertex of H, G is a bloomed C_k with trivial blooms. ∎

We can prove now the main result of this section.

Corollary 13. Let G be a paw-free graph. Then, G is a CA graph if and only if G contains no induced bipartite claw, $K_{2,3}$, G_2, G_4, G_7, or C_j^*, for any $j \geq 4$.

Proof. Let H be a paw-free graph. Suppose, by way of contradiction, that H is not isomorphic to the bipartite claw, $K_{2,3}$, G_2, G_4, G_7, or C_j^*, for $j \geq 4$, but H is still a minimally non-CA graph. Since H is paw free, H is non-basic and, by Corollary 5, H contains an induced C_j for some $j \geq 4$. By Theorem 11 and Theorem 12, H is either an augmented multiple of $\overline{tK_2}$ for some $t \geq 2$ or a bloomed C_j with trivial blooms. It is easy to see that a bloomed C_j with trivial blooms is CA, and an augmented multiple of $\overline{tK_2}$ is shown to be CA in Theorem 8. In both cases, we get a contradiction. ∎

4.3. Claw-free chordal graphs

A graph is *claw-free chordal* if it contains neither an induced claw nor a hole. Claw-free graphs are widely studied in the literature, see for example [15] or recent results in [3]. Besides, as PCA graphs are claw-free, the study of claw-free chordal graphs arises naturally from the characterization of PCA graphs within the class of chordal graphs.

Lemma 1. Let G be a {claw,net*,G_5,G_6}-free chordal graph that contains a net L induced by $\{t_1, t_2, t_3, b_1, b_2, b_3\}$, where $\{t_1, t_2, t_3\}$ induces a triangle and b_i is adjacent to t_i for $i = 1, 2, 3$. If v is a vertex in $G - L$, then $N_L(v)$ is either $\{b_i, t_i\}$, or $\{t_1, t_2, t_3, b_i\}$ or $\{b_{i+1}, t_{i+1}, t_{i+2}, b_{i+2}\}$, for some $i \in \{1, 2, 3\}$ (indices should be understood modulo 3).

Proof. We will analyze the different cases for $|N_L(v)|$. If $|N_L(v)| = 0$, then $L \cup \{v\}$ induces net*, a contradiction. If $|N_L(v)| = 1$, then either $N_L(v) = \{b_i\}$ or $N_L(v) = \{t_i\}$ for some $i \in \{1, 2, 3\}$. In the first case, $L \cup \{v\}$ induces G_5; in the second case, b_i, t_i, t_{i+1}, v induce a claw. In both cases, we get a contradiction.

If $|N_L(v)| = 2$, then the representative cases for $N_L(v)$ up to symmetry are: $\{b_i, b_{i+1}\}$, $\{t_i, t_{i+1}\}$, $\{b_i, t_{i+1}\}$, $\{b_i, t_i\}$. In the first case, $b_i t_i t_{i+1} b_{i+1} v$ is a hole; in the second and third cases, $t_{i+1}, t_{i+2}, b_{i+1}, v$ induce a claw. So, if $|N_L(v)| = 2$, then $N_L(v) = \{b_i, t_i\}$ for some $i \in \{1, 2, 3\}$.

If $|N_L(v)| = 3$, then the representative cases up to symmetry are: $\{b_1, b_2, b_3\}$, $\{b_i, b_{i+1}, t_{i+2}\}$, $\{t_1, t_2, t_3\}$, $\{b_i, b_{i+1}, t_{i+1}\}$, $\{b_i, t_{i+1}, t_{i+2}\}$, $\{b_i, t_i, t_{i+1}\}$. In the first two cases, $N_L(v) \cup \{v\}$ induces a claw; in the third case, $N_L(v) \cup \{v\}$ induces G_6; in the fourth an fifth cases, $b_i t_i t_{i+1} v$ is a hole; in the last case $t_{i+1}, b_{i+1}, t_{i+2}, v$ induce a claw. In all the cases we get a contradiction.

Finally, if v is adjacent to b_{i+1}, b_{i+2} and to either b_i or t_i, then $\{v, b_{i+1}, b_{i+2}, b_i\}$ or $\{v, b_{i+1}, b_{i+2}, t_i\}$ induces a claw, respectively. So, if $|N_L(v)| \geq 4$, then $N_L(v)$ is either $\{t_1, t_2, t_3, b_i\}$ or $\{b_{i+1}, t_{i+1}, t_{i+2}, b_{i+2}\}$, for some $i \in \{1, 2, 3\}$. ∎

Theorem 14. Let G be a claw-free chordal graph that contains an induced net, and such that all its proper induced subgraphs are CA graphs. Then, exactly one of the following conditions holds:

(1) G is isomorphic to net*, G_5 or G_6.

(2) G is a CA graph.

Proof. Assume that G is not isomorphic to net*, G_5 or G_6. Since these graphs are non-CA and all proper induced subgraphs of G are CA, G contains no induced net*, G_5 or G_6.

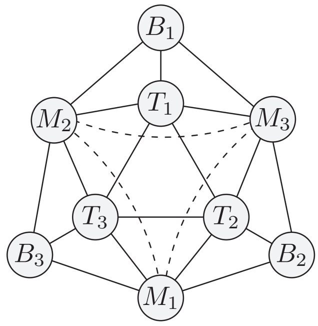

FIGURE 3. Circles represent cliques. Two circles are adjacent, non-adjacent or joined by a dotted line if the corresponding cliques are mutually complete, anticomplete, or not necessarily complete or anticomplete, respectively.

We claim that G has as an induced subgraph H that is a multiple of a net; i.e., the vertices of H can be partitioned into six non-empty cliques $B_1, B_2, B_3, T_1, T_2, T_3$ such that T_1, T_2, T_3 are mutually complete and T_i is complete to B_i and anticomplete to B_{i+1} and B_{i+2}, for each $i = 1, 2, 3$ (from now on, the indices should be understood modulo 3). Moreover, the vertices of $G - H$ can be partitioned into three (possibly empty) cliques M_1, M_2, M_3 such that, for each $i = 1, 2, 3$, M_i is complete to B_{i+1}, B_{i+2}, T_{i+1} and T_{i+2} and anticomplete to B_i and T_i. A scheme of this situation can be seen in Figure 3.

We will prove the claim by induction on the number n of vertices of G. Clearly, if G is a net, then the claim holds. Assume that $n > 6$ and that the desired result holds for graphs with less than n vertices. Since $n > 6$, there exists a vertex v of G such that $G' = G - \{v\}$ contains an induced net. By inductive hypothesis, since G' is claw-free chordal, G' has an induced subgraph H that is a multiple of a net and the vertices of $G' - H$ can be partitioned into three cliques M_1, M_2, M_3 satisfying the conditions above.

Choose $t_i \in T_i$, $b_i \in B_i$ for each $i = 1, 2, 3$ (recall that T_i and B_i are non-empty for $i = 1, 2, 3$). Let L be the subgraph induced by $\{t_1, t_2, t_3, b_1, b_2, b_3\}$. By Lemma 1, either $N_L(v) = \{b_i, t_i\}$, $N_L(v) = \{t_1, t_2, t_3, b_i\}$ or $N_L(v) = \{b_{i+1}, t_{i+1}, t_{i+2}, b_{i+2}\}$, for some $i \in \{1, 2, 3\}$.

Suppose first that $N_L(v) = \{t_i, b_i\}$ for some $i \in \{1, 2, 3\}$. Let $j \in \{1, 2, 3\}$, $b'_j \in B_j$, and L' be the net induced by $\{t_1, t_2, t_3, b'_j, b_{j+1}, b_{j+2}\}$. Applying Lemma 1 to L', it follows that v is adjacent to b'_j if and only if $j = i$. Thus, v is complete to B_i and anticomplete to B_{i+1} and B_{i+2}. Using the same strategy, we can prove that v is complete to T_i and anticomplete to T_{i+1} and T_{i+2}. Since G is claw-free, v must be complete to M_{i+1} (if w were a non-neighbour of v in M_{i+1}, then t_i, t_{i+1}, w, v would induce a claw) and, by symmetry, v is also complete to M_{i+2}. Moreover, since G is C_4-free, v is anticomplete to M_i (if w were a neighbour of v in M_i, then t_i, t_{i+1}, w, v would induce C_4). Thus, the claim holds for G replacing B_i by $B_i \cup \{v\}$.

Next, suppose that $N_L(v) = \{t_1, t_2, t_3, b_i\}$ for some $i \in \{1, 2, 3\}$. Reasoning as in the first case, it follows that v is complete to T_1, T_2, T_3, B_i and anticomplete to B_{i+1} and

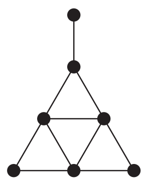

FIGURE 4. The graph S.

B_{i+2}. Since G is C_4-free, v must be complete to M_{i+1} (if w were a non-neighbour of v in M_{i+1}, then b_i, v, t_{i+2}, w would induce a C_4) and, by symmetry, also to M_{i+2}. Since G is claw-free, v must be anticomplete to M_i (if w were a neighbour of v in M_i, then w, b_{i+1}, b_{i+2}, v would induce a claw). Thus, the claim holds for G replacing T_i by $T_i \cup \{v\}$.

Finally, suppose that $N_L(v) = \{b_{i+1}, t_{i+1}, t_{i+2}, b_{i+2}\}$ for some $i \in \{1, 2, 3\}$. Reasoning again as in the first case, it follows that v is complete to B_{i+1}, T_{i+1}, T_{i+2}, B_{i+2} and anticomplete to B_i and T_i. Since G is claw-free, v must be complete to M_i (if w were a non-neighbour of v in M_i, then t_i, t_{i+1}, w, v would induce a claw). Thus, the claim holds for G replacing M_i by $M_i \cup \{v\}$. This ends the proof of the claim.

If M_i and M_{i+1} are non-empty and m_i, m_{i+1} are vertices in M_i and M_{i+1}, respectively, then either $m_i t_{i+1} t_i m_{i+1} b_{i+2}$ induce a C_5 or $m_i t_{i+1} t_i m_{i+1}$ induce a C_4. Since G is chordal, at most one of $\{M_1, M_2, M_3\}$ is non-empty. Consequently, G is either a multiple of a net (if every M_i is empty) or a multiple of the graph S depicted in Figure 4. Since the net and S are easily seen to be a CA graph, G is also a CA graph. ∎

We can now prove the main result of this section.

Corollary 15. Let G be a claw-free chordal graph. Then, G is CA if and only if G contains no induced tent*, net*, G_5 or G_6.

Proof. Let H be a claw-free chordal graph. Suppose, by way of contradiction, that H is not isomorphic to tent*, net*, G_5 or G_6, but H is still a minimally non-CA graph. Since H is claw-free and chordal, H is non-basic and, by Corollary 5, H contains an induced net or tent. If H contains an induced net, then by Theorem 14, H is isomorphic to a net*, G_5 or G_6, a contradiction. Thus, H contains no induced net but an induced tent. Since H is non-basic, it is connected (Corollary 4). So, by Theorem 1, H is a multiple of a tent and, in particular, a CA graph, a contradiction. ∎

4.4. Diamond-free graphs

A *diamond-free* graph is a graph with no induced diamond. Diamond-free graphs have been extensively studied. (See, for example, [2, 4, 19].)

Theorem 16. Let G be a diamond-free graph that contains a hole, and such that all its proper induced subgraphs are CA graphs. Then, exactly one of the following conditions holds:

(1) G is isomorphic to $K_{2,3}$, G_2, G_3, G_4, G_7, $\overline{C_6}$, G_9, or C_j^* for some $j \geq 4$.
(2) G is a CA graph. More precisely, if H is any induced hole of G, and $V(H) = \{h_1, \dots, h_k\}$ where h_i is adjacent to h_{i+1} for each $i = 1, \dots, k$ (indices should be understood modulo k), then the vertices of $G - H$ can be partitioned into $k+1$ (possibly empty) pairwise anticomplete sets $U_1, \dots, U_k, S$ such that the following conditions hold:

- For each $i = 1, \dots, k$, $G[U_i]$ is the union of vertex-disjoint cliques and for each $u \in U_i$, $N_H(u) = \{h_i\}$.
- For each $s \in S$ there is an integer i, $1 \leq i \leq k$, such that $N_H(s) = \{h_i, h_{i+1}\}$; in addition, if $s_1, s_2 \in S$, then s_1 and s_2 are adjacent if and only if $N_H(s_1) = N_H(s_2)$.

Proof. Assume that G is not isomorphic to $K_{2,3}$, G_2, G_3, G_4, G_7, $\overline{C_6}$, G_9, or C_j^* for any $j \geq 4$. Since all of these graphs are non-CA and all proper induced subgraphs of G are CA, G contains none of these graphs as induced subgraphs.

Let H be an induced hole on G of length k and let v be any vertex of $G - H$. Since G is not isomorphic to $K_{2,3}$, G_2, G_3, G_4, or C_j^*, for any $j \geq 4$, by Theorem 6, either v is complete to H or $N_H(v)$ induces a non-empty path in H. Since G is diamond-free, v is adjacent to at most two vertices of H. So, each vertex of $G - H$ is adjacent to either a single vertex or two adjacent vertices of H.

Let $V(H) = \{h_1, \dots, h_k\}$, where h_i is adjacent to h_{i+1} for each $i = 1, \dots, k$ (from now on, indices should be understood modulo k). Let U_i be the set of vertices v of $G - H$ with $N_H(v) = \{h_i\}$. Since h_i is adjacent to all vertices of U_i and G is diamond-free, $G[U_i]$ contains no induced P_3 and therefore $G[U_i]$ is the union of vertex disjoint cliques.

We now show that if $i \neq j$, then U_i is anticomplete to U_j. Suppose, by way of contradiction, that there exist i and j, $i \neq j$, such that some vertex $u_i \in U_i$ is adjacent to some vertex $u_j \in U_j$. Let P^1 and P^2 be the two distinct paths on H joining h_i and h_j. If P^1 has more than four vertices, then there is an interior vertex w of P^1 that is anticomplete to P^2, so $\{u_i, u_j\} \cup V(P^2) \cup \{w\}$ induces on G a graph isomorphic to C_m^* for some $m \geq 4$, a contradiction. Thus, each one of P^1 and P^2 has at most four vertices. Without loss of generality, we may assume that $|P^1| \leq |P^2|$. If $|P^1| = 2$ and $|P^2| = 4$, then $\{u_i, u_j\} \cup V(H)$ induces G_7; if $|P^1| = 3$ and $|P^2| = 3$, then $\{u_i, u_j\} \cup V(H)$ induces G_2; if $|P^1| = 3$ and $|P^2| = 4$, then $\{u_i, u_j\} \cup V(H)$ induces G_4; if $|P^1| = 4$ and $|P^2| = 4$, then $\{u_i, u_j\} \cup (V(H) - \{h_i\})$ induces a bipartite claw. In all the cases, we get a contradiction. We conclude that if $i \neq j$, then U_i is anticomplete to U_j.

Let S be the set of vertices v of $G - H$ that are adjacent to two vertices of H. Let s_1, s_2 be two vertices of S, i and j be such that $N_H(s_1) = \{h_i, h_{i+1}\}$ and $N_H(s_2) = \{h_j, h_{j+1}\}$. Since G is diamond-free, if $i = j$, then s_1 and s_2 must be adjacent and if $|i - j| = 1$, then s_1 and s_2 must be non-adjacent. Suppose now that $|i - j| > 1$, so h_i, h_{i+1}, h_j

and h_{j+1} are pairwise distinct. Assume for contradiction that s_1 and s_2 are adjacent. Let P^1 be the path on H whose vertices are $\{h_{i+1}, h_{i+2}, \ldots, h_j\}$ and P^2 be the path on H whose vertices are $\{h_{j+1}, h_{j+2}, \ldots, h_i\}$. If P^1 and P^2 have no internal vertices, then $\{s_1, s_2\} \cup V(H)$ induces $\overline{C_6}$, a contradiction. We can assume, without loss of generality, that P^1 has at least one internal vertex w. But, then w is anticomplete to the hole induced on G by $\{s_1, s_2\} \cup V(P^2)$, hence $\{s_1, s_2, w\} \cup V(P^2)$ induces on G a graph isomorphic to C_m^* for some $m \geq 4$, a contradiction. So, s_1 and s_2 are non-adjacent.

Now we will prove that U_i is anticomplete to S for each $i = 1, \ldots, k$. Suppose, by way of contradiction, that there exist adjacent vertices $u_i \in U_i$ and $s \in S$, and let j be such that $N_H(s) = \{h_j, h_{j+1}\}$. Since G is diamond-free, i is different from j and $j+1$. Let P^1 be the path on H whose vertices are $\{h_i, h_{i+1}, \ldots, h_j\}$ and P^2 the path on H whose vertices are $\{h_{j+1}, \ldots, h_{i-1}, h_i\}$. If P^2 has more than three vertices, then h_{j+2} is anticomplete to the hole induced by $\{s, u_i\} \cup V(P^1)$, a contradiction. Analogously, P^1 has at most three vertices. If $|P^1| = 2$ and $|P^2| = 3$, then $\{u_i, s\} \cup V(H)$ induces G_3; if $|P^1| = 3$ and $|P^2| = 3$, then $\{u_i, s\} \cup V(H)$ induces G_9. We may assume $|P_1| \leq |P_2|$. In both cases, we have a contradiction. We conclude that U_i is anticomplete to S for each $i = 1, \ldots, k$.

Finally, it is not difficult to see that a graph satisfying these conditions is a CA graph. This concludes the proof. ∎

Theorem 17. Let G be a diamond-free chordal graph that contains an induced net, and such that all its proper induced subgraphs are CA graphs. Then, exactly one of the following conditions holds:

(1) G is isomorphic to a net*, G_5, or G_6.
(2) G is a fully bloomed triangle, and in consequence, it is a CA graph.

Proof. Assume that G is not isomorphic to net*, G_5, or G_6. Since all of these graphs are non-CA and all proper induced subgraphs of G are CA, G contains none of these graphs as induced subgraphs.

We will show that G is a fully bloomed triangle, and, as a consequence, a CA graph. We will argue by induction on the number of vertices of G.

Clearly, a net is a fully bloomed triangle. Suppose that G has $n > 6$ vertices and that the result holds for graphs with $n-1$ vertices. Since G has more than six vertices, there exists a vertex v of G such that $G - \{v\}$ contains an induced net.

Moreover, $G - \{v\}$ is diamond-free chordal, all its proper induced subgraphs are CA graphs and it is not isomorphic to net*, G_5, or G_6. So, by inductive hypothesis, $G - \{v\}$ is a fully bloomed triangle. That is, there exists a triangle T of $G - \{v\}$ such that the remaining vertices of $G - \{v\}$ induce a disjoint union of complete graphs $M_1, M_2, \ldots, M_m$, where each M_i is complete to one vertex of T and anticomplete to the others, and each vertex of T is complete to at least one of $M_1, M_2, \ldots, M_m$. The vertex v is adjacent to at least one vertex of $G - \{v\}$ because G contains no induced net*. On the other hand, since G is chordal and diamond-free and $G - \{v\}$ is connected, $N(v)$ induces a complete graph on G. So, either $N(v) \subseteq T$ or $N(v) \subseteq M_i \cup \{t\}$, where $t \in T$ and M_i is a bloom complete to t. In the first case, since G contains no induced G_6, $|N(v)| \neq 3$, and since G is diamond free, $|N(v)| \neq 2$. Therefore, $N(v) = \{t\}$ for some $t \in T$ and $\{v\}$ is a new bloom complete to t. In the second case, since G is diamond-free, either $|N(v)| = 1$ or $N(v) = M_i \cup \{t\}$. If $N(v) = \{t\}$ with $t \in T$, then $\{v\}$ is a new bloom for t; if $N(v) = \{w\}$ with $w \in M_i$, then G contains an induced G_5, a contradiction; if $N(v) = M_i \cup \{t\}$, then G is a fully bloomed triangle replacing M_i by $M_i \cup \{v\}$. ∎

Finally, we can prove the main result of this section.

Corollary 18. A diamond-free graph G is CA if and only if G contains no induced bipartite claw, net*, $K_{2,3}$, G_2, G_3, G_4, G_5, G_6, G_7, $\overline{C_6}$, G_9, or C_j^*, for any $j \geq 4$.

Proof. Let H be a diamond-free graph. Suppose, by way of contradiction, that H is not isomorphic to the bipartite claw, net*, $K_{2,3}$, G_2, G_3, G_4, G_5, G_6, G_7, $\overline{C_6}$, G_9, or C_j^*, for any $j \geq 4$, but H is still a minimally non-CA graph. Since H is not an interval graph but it does not contain a bipartite claw and it is diamond-free, by Theorem 3, H contains either a hole or an induced net. If H contains a hole, H contradicts Theorem 16. Otherwise, H is chordal. Then, H contains an induced net, and so H contradicts Theorem 17. ∎

5. SUMMARY AND FURTHER RESULTS

The partial characterizations of circular-arc graphs by forbidden induced subgraphs obtained in this work are summarized in Table I.

TABLE I. Minimal forbidden induced subgraphs for circular-arc graphs in each studied class.

Graph classes	**Minimal forbidden induced subgraphs**	**Reference**
P_4-free graphs	$K_{2,3}$, C_4^*	§ 4.1.
Paw-free graphs	bipartite claw, $K_{2,3}$, G_2, G_4, G_7, C_j^* $(j \geq 4)$	§ 4.2.
Claw-free chordal graphs	tent*, net*, G_5, G_6	§ 4.3.
Diamond-free graphs	bipartite claw, net*, $K_{2,3}$, G_2, G_3, G_4, G_5, G_6, G_7, $\overline{C_6}$, G_9, C_j^* $(j \geq 4)$	§ 4.4.

A CA graph is a *normal circular-arc* (NCA) graph if it admits a circular-arc model such that no two arcs cover the whole circle. For example, interval graphs and semi-circular graphs are NCA graphs. An example of a graph which is not NCA is given in Figure 5. This concept was studied in [8, 9, 10], but the terminology NCA was introduced in [12]. The characterization of non-NCA graphs by minimal forbidden induced

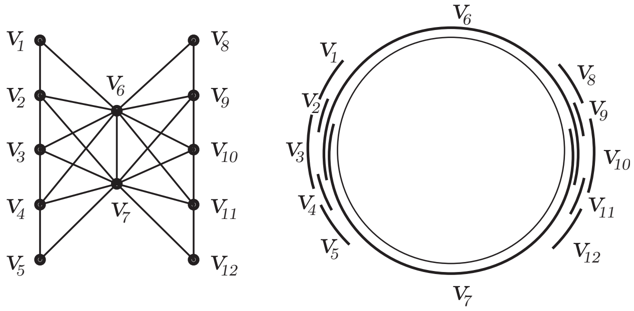

FIGURE 5. Minimally non-NCA graph that is CA, and its circular-arc model.

subgraphs is not known. The proofs in this paper show that, for the classes analyzed here, all CA graphs are also NCA. So, the characterizations obtained for CA graphs also hold for NCA graphs. Moreover, we can state the following result.

Corollary 19. If H is a minimally non-NCA graph and H is a CA graph, then H contains an induced diamond, an induced P_4, an induced paw, and either an induced claw or a hole.

ACKNOWLEDGMENTS

This work was partially supported by UBACyT Grant X069, Argentina. The first author was partially supported by UBACyT Grant X606, Argentina and CNPq under PROSUL project Proc. 490333/2004-4, Brazil. The second author was partially supported by FONDECyT Grant 1080286 and Millennium Science Institute “Complex Engineering Systems”, Chile and CNPq under PROSUL project Proc. 490333/2004-4, Brazil. The authors are grateful to Javier Marenco and the anonymous referees for their many suggestions that improved the presentation of this paper.

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