Dynamics of a minority game with an additional layer of interaction

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Abstract

We analyze the variant of the minority game with an additional interaction mechanism introduced by I. Caridi and H. Ceva in [3], which considers a periodic square lattice of agents where some of them may share information with their neighbors. We show that low levels of interaction in this model induce the interacting agents to play with their most similar strategies (those minimizing the Hamming distance between them), hence the resulting dynamics can be replicated by introducing groups of similar one-strategy agents. We also study the reaction of non-interacting agents to the global perturbations introduced by the interacting agents, showing the emergence of cluster-detection patterns.

Key words: minority game, interacting agents
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1 Introduction

The standard *minority game* (MG) is a discrete adaptive model based on the "El Farol" bar problem [1], which was introduced in [2] in order to study strategical behaviors. In this model, N(odd) agents must independently choose between two actions (usually denoted by 0 and 1), and the agents who make the minority decision win. Each agent's choice depends on a set of s strategies. Each strategy predicts the next winning action (0 or 1) by processing the outcomes from the last m time steps, which is the only available public information. The value m is known as the agents' *memory*. Since every strategy contains the 2^m possible historic states, the whole pool has 2^{2^m} strategies; and

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at the beginning of the game each agent randomly draws her set of s of them (maybe repeated by chance). At each step, the strategies that have correctly predicted the outcome are rewarded one point, regardless of usage. Far away from the possibility of knowing her maximization-of-payoff choice, each agent is confined to play in the way her own best performing strategy (i.e., the one from her set with the best score up to that moment) suggests.

The MG is associated with the analysis of simple financial markets due to the similar kind of binary decisions that must be made (e.g., "buy" or "sell") and to the bounded rationality and incomplete information that the individuals have. However, some other features may be taken into account in order to deal with more complex instances. In particular, several models introduce local information mechanisms, by which some agents get information from some other agents (e.g., their next bet) in order to make better predictions of the game's future outcomes [3–8].

In this paper we focus on the dynamics of the interesting model introduced by I. Caridi and H. Ceva in [3], which is based on the original MG but includes local interaction as a mechanism for sharing information. In this model, the N agents are distributed on a $k \times k$ square periodic lattice (where $N = k^2$), and at the beginning of the game a fraction $p \in [0, 1]$ of them is designated to be *interacting agents* (IAs). Therefore, every IA can have from zero to four neighboring IAs in the grid which, in turn, may have further IAs as neighbors. At each time step the agents follow the usual rules of the MG, but the IAs are given one extra opportunity to change their bets, after knowing what their neighboring IAs will do in the same step. At this stage, every IA queries her neighboring IAs by looking at what their best strategy has advised them to do and, after this survey, the IA will choose to be in the minority of the group composed by her interacting neighbors plus herself. Hence, if more than half of her neighbors (plus herself) choose one side, then the IA will chose the other one, regardless of what her best strategy suggested. If half of the neighbors (plus herself) choose each side, then the IA will just follow her best strategy. Once all the agents have made their choice, they make their moves simultaneously and the minority side is obtained as usual. Points are awarded as in the standard MG, but whenever a player wins because she plays against the prediction of her best strategy in order to avoid the majority of her local neighborhood, then no points are added to her strategies.

I. Caridi and H. Ceva study in [3] the behaviour of the *reduced variance* σ^2/N of the time series of the number of agents belonging to the bet 1 for this model in terms of the usual scale variable $z = 2^m/N$ for different values of p. As a result they find, compared with the MG, on one hand, an improved resource distribution for small values of p ($\sigma^2/N \cong 0.03$ against $\sigma^2/N \cong 0.04$ for the MG [9]) and, on the other hand, a smaller value of z for which σ^2/N attains its minimum ($z \cong 0.3$ against $z \cong 0.45$ for the MG [9]). Moreover, by computing the reduced variance of even and odd steps along the voting process they show that, in the informationally efficient region (i.e., when small values of z are considered), a period-two dynamics of the MG can be found; that is, dynamics in which during the odd appearance of a given history (binary string) the resulting outcome is random, but in its even appearance the outcome is the opposite to the previous one [3,9]. Finally, by measuring the entropy rate of the outcome they observe that the voting evolution falls in cyclic motions for $p > p^*$ with $p^* \sim 0.5$.

In this work we show that the local interaction mechanism introduced by I. Caridi and H. Ceva encourages the formation of groups of agents with a concerted behavior derived from the sharing of information, which act as groups of similar single-strategy agents. This fact leads to a better understanding of the underlying dynamics, in particular providing a simple explanation for the backward movement of the minimum value in the reduced variance plot reported in [3].

This paper is organized as follows. On one hand, Section 2 studies the voting dynamics of IAs, showing how this behavior can be replicated by single-strategy agents. On the other hand, Section 3 explores the reaction of non-interacting agents to the global effects generated in the game by the IAs. Section 4 closes the paper with some concluding remarks. For agents with more than one strategy, we fix $s = 2$ throughout this paper.

2 Dynamics of interacting agents

We define the *interaction graph* to be the graph $G = (V, E)$, where the vertex set V corresponds to the set of IAs and two vertices are joined by an edge in E if the associated IAs are the nearest neighbors in the square lattice. A

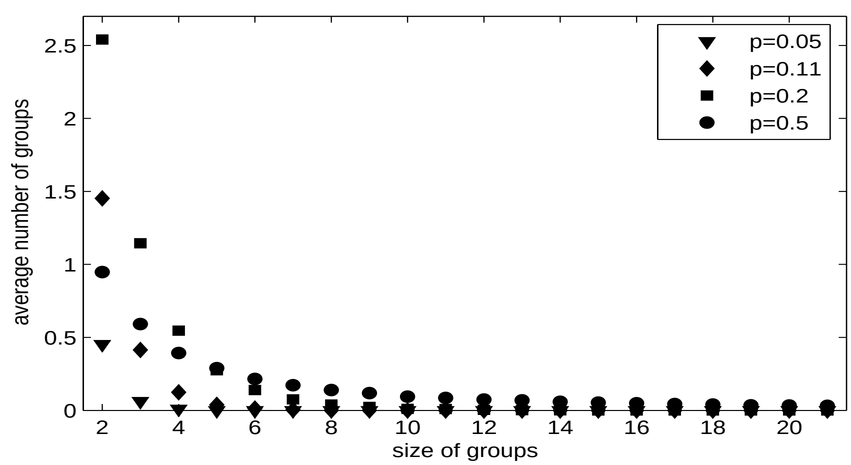

Fig. 1. Number of groups of IAs as a function of their sizes. Points are averages over 10,000 outcomes of 11×11 periodic square lattices.

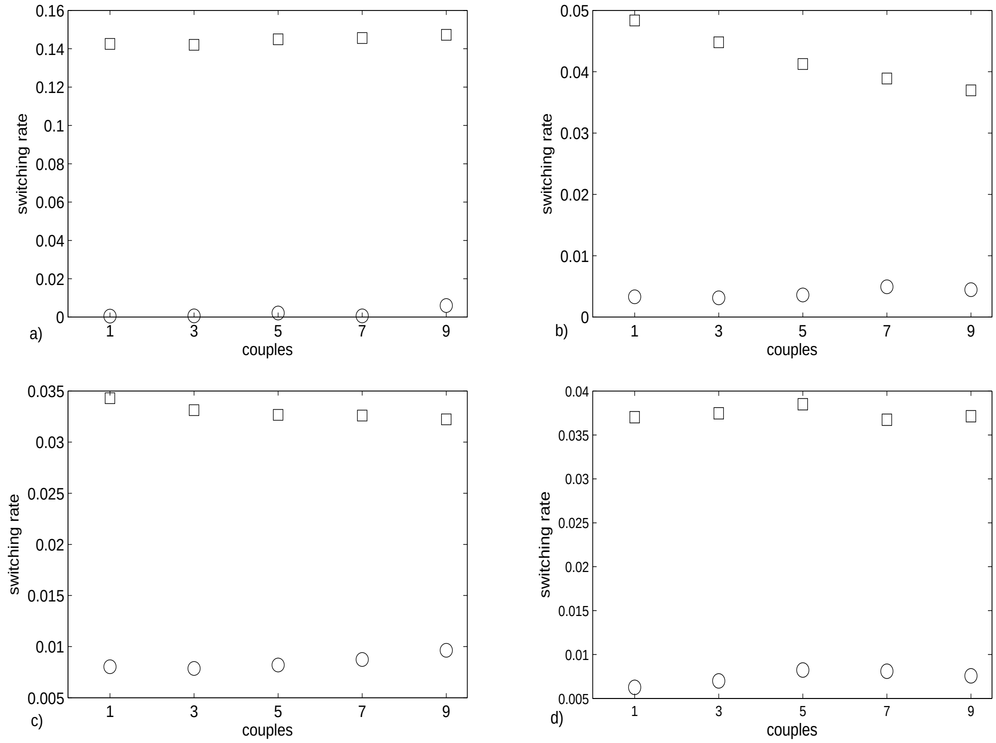

Fig. 2. Switching rate of independent agents (□) and of couples of IAs (○) as a function of couples of IAs. Points are averages over 100 outcomes of 11×11 periodic square lattices after 50,000 time steps with a) $m = 2$, b) $m = 5$, c) $m = 8$ and d) $m = 11$.

connected component of G is called a *group* of IAs, and the number of IAs in the group is called the *size* of the group. Figure 1 shows the average number of groups as a function of their sizes for several coefficients p of local information, including $p \sim 0.11$, the optimal coefficient according to [3]. There is a relatively high number of groups of size 2, hence we now study the voting dynamics in such two-agent groups.

In a group of two IAs there are two alternatives according to what the best strategies of both agents say: either to play the same or to play differently. In the first case, if the corresponding best strategies suggest the same side, then the extra opportunity of these IAs will force them to swap their bets. Hence they will play the same anyhow but the strategies will not get any virtual points. The score of their strategies will not change and, therefore, both IAs will repeat the same strategies in the next step. In the second case (i.e., if the two IAs play differently), then they will not change their initial choices in the interaction phase, hence one of them will win and the other will not. The strategy used by the infortunate IA will sum no virtual points but the non-used strategy could have rightly predicted the game's outcome and it will get one additional virtual point, so it may become the preferred strategy.

These dynamics clearly force the two IAs to play with their most similar strategies: in case of coincidence these strategies do not get any points, hence

m	Similarity rate
2	0.742
3	0.677
4	0.627
5	0.591
6	0.565
7	0.544

m	Similarity rate
8	0.532
9	0.523
10	0.516
11	0.512
12	0.508
13	0.504

Table 1
Similarity rates of two strategies as a function of m; i.e., the expected number of coincident positions between the two most similar strategies of any two agents. Values are obtained by averaging the fraction of coincidences over 10,000 couples of pairs of strategies.

ensuring the repetition of both strategies in the next step. In other words, since each player has two strategies, then two players have four possible combinations of strategies, but the most used combination will be the one composed by those strategies with the highest number of coincident predictions. This behavior produces more coincidences between two neighboring IAs than the expected coincidences for a couple of non-interacting agents. In particular, the switching rate (i.e., the average number of steps in which a player switches to another strategy divided by the total number of steps [2]) is expected to be lower for IAs in two-agent groups than for standard players, regardless of their final score. Our computational experience confirms this behavior, and these results are presented in Figure 2.

The previous observations also imply that we can approximate a couple of IAs by two agents with only one strategy each, such that these two strategies coincide in a certain number of positions. If $v, w \in \{0,1\}^{2^m}$ are two strategies, we denote by $c(v,w)$ the number of coincident positions between them, i.e., $c(v,w) = 2^m - \text{dist}(v,w)$, where $\text{dist}(v,w)$ denotes the Hamming distance between v and w. If w_1^i and w_2^i are the two strategies of agent i, we define

$$d(i,j) \;=\; \frac{max\{c(w_1^i,w_1^j),c(w_1^i,w_2^j),c(w_2^i,w_1^j),c(w_2^i,w_2^j)\}}{2^m}.$$

Finally, we define the *similarity rate* to be the expected value of $d(i,j)$ for randomly-chosen two-strategy agents i and j (see Table 1 for a computation of these values). The similarity rate represents the expected number of coincident positions between the two most similar strategies of any two agents.
We replace, therefore, each two-agent group by two single-strategy non-interacting agents (called *probabilistic agents*), such that each position of these two strategies coincides with probability equal to the corresponding similarity rate. According to the previous discussion, this replacement is intended to replicate the behavior of the group. A comparison of the efficiency of the

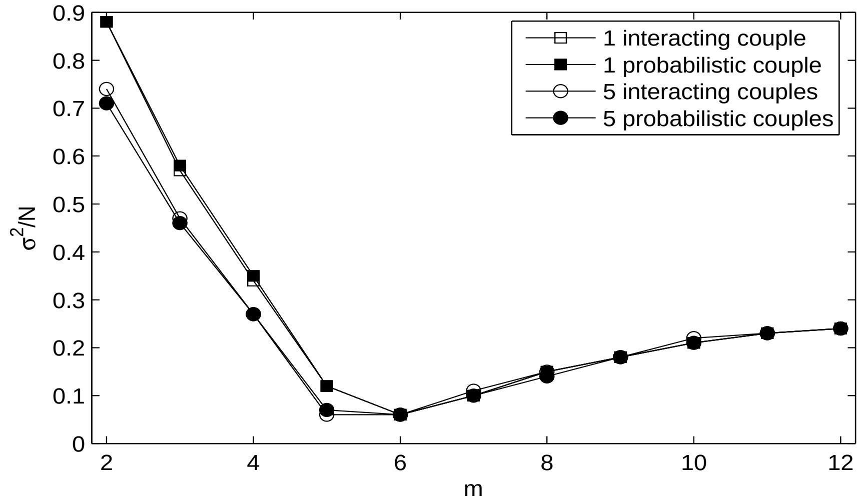

Fig. 3. Reduced Variance (σ^2/N) as a function of memory (m) comparing different number of interacting couples with their equivalent probabilistic couples. Points are averages over 100 outcomes of 11×11 periodic lattices after 50,000 time steps.

original mechanism by I. Caridi and H. Ceva with couples of IAs on one hand and the model with couples of probabilistic agents on the other hand is presented in Figure 3. A close correlation between these two different models is clearly observed, implying that the new probabilistic agents properly model the behavior of couples of IAs in the original game. This shows that the interaction procedure proposed in [3] can be simply understood as a mechanism for the introduction of repetition patterns within the strategies of the agents designated to be IAs.

The previous discussion applies to groups of IAs of size 2 but, in addition, there may be groups of larger sizes. A properly defined similarity rate for such groups is much smaller than the similarity rate for two-agent groups, due to the increasing difficulty of synchronizing strategies in order to achieve coincident bets. This fact, combined with the small expected number of groups with more than two agents, implies that two-agent groups are responsible for most of the effects introduced in the game by the local interaction mechanism.

3 The reaction of non-interacting agents

In this section we study the reaction of non-interacting agents to the perturbations introduced in the game by the IAs. As the previous section shows, the IAs in two-agent groups tend to play with their most similar strategies, hence their best performing strategies will make coincident predictions with a high probability. A similar situation may hold in groups of IAs with more than two agents, but their effect is diminished by the small expected number

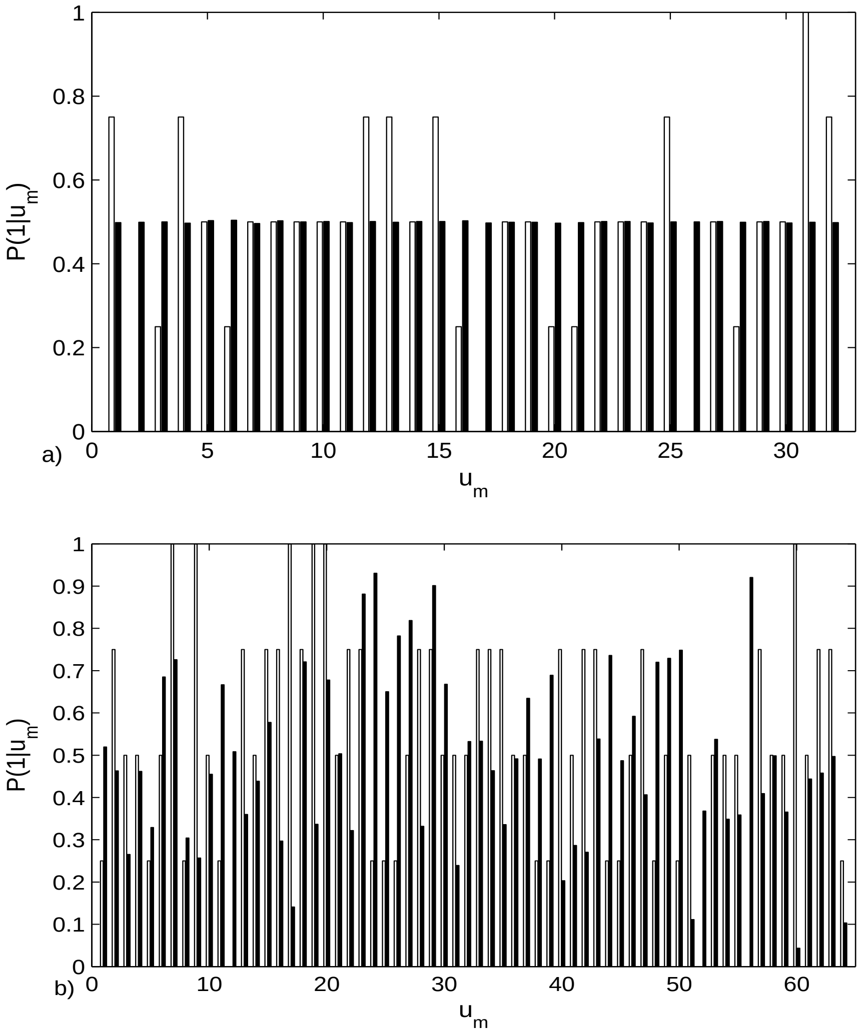

Fig. 4. a) Histogram of the conditional probabilities $P_c(1|u_m)$ (white bars) and $P_i(1|u_m)$ (black bars) with $m = 5$ for 117 individual 2-strategy agents plus two couples of probabilistic agents (i.e., the agents from each couple have similar strategies). There are 32 possible combinations of 0's and 1's. The numbers, when written in binary form, yield the strings u_m. b) Histogram of the conditional probability $P_c(1|u_m)$ (white bars) and $P_i(1|u_m)$ (black bars) with $m = 6$ for 117 individual 2-strategy agents plus two couples of probabilistic agents with one similar strategy. There are 64 possible combinations of 0's and 1's. In both figures, bars are averages over 100 outcomes of 11×11 periodic lattices after 50,000 time steps.

of such groups and the low similarity rates within them. We define a *cluster* to be a subset of a group of IAs, whose agents coincide in their bets with a higher probability than regular non-interacting agents. Note that, due to the discussion in the previous section, any two-agent group is also a two-agent cluster.

Since the agents in a cluster tend to make coincident predictions, then clusters produce larger values of σ^2/N easily detected by the other agents and, moreover, this detection is easier as the number of clusters increases. As the IAs who used the extra opportunity do not sum points to their strategies regardless of prediction, the probability for agents of a cluster of repeating their bets for two subsequent occurrences of a given string is larger than the standard value of $(2^m)^{-1/2}$ [9]. This allows other agents to recognize regular clusters as weighted-like agents, hence learning to take clusters into account for each decision. Since these deviations increase with the number of clusters, individual players "see" the game earlier than in the MG, hence the value of m attaining the minimum σ^2/N is smaller for this model, as shown in Figure 3 and in [3].

The detection of the interacting agents by independent ones can be noticed by studying the outcome of the game. To this end, we compute the conditional probability of playing 1 for a given string u_m of length m when couples of probabilistic agents take part in the game. Figure 4.a shows these values $P_i(1|u_m)$ and $P_c(1|u_m)$ for $m = 5$ for individual players and two couples of probabilistic agents with similar strategies (similarity rate 0.591), respectively. The histogram for $P_i(1|u_m)$ is flat for this m (as seen in [9] for the standard MG) showing that the clusters have not been detected yet. Figure 4.b shows the values $P_i(1|u_m)$ and $P_c(1|u_m)$ for $m = 6$, and now individual players tend to avoid the prediction made by the clusters.

In order to measure this tendency we compute the average distance between $P_c(1|u_m)$ and $P_i(1|u_m)$ by

$$D = \frac{1}{2^m} \sum_{u_m} |P_i(1|u_m) - P_c(1|u_m)|,$$

and Figure 5 shows these measurements as a function of m. As we can see, for $m \leq 5$ the plot of $P_i(1|u_m)$ is flat ($P_i(1|u_m) \sim 0.5$) and the expected difference appears to be $|P_i(1|u_m) - P_c(1|u_m)| \sim |0.5 - P_c(1|u_m)| \sim 0.188$. On the other hand, cluster detection appears for $m \geq 6$, as shown by larger average distances for growing values of m.

4 Concluding remarks

In this paper we studied the dynamics of a particular MG model with an additional layer of interaction over a square grid. Regardless of the complexity of the instance, local information managed by agents suggests that the resulting dynamics of the game can be understood as an MG where groups of similar one-strategy agents are allowed, the similarity rate being a measure of the level of regular clusterization introduced by the proposed interaction. As a consequence, agents learn sooner and the game moves the memory size corresponding to the minimum waste of resources backwards as the number of

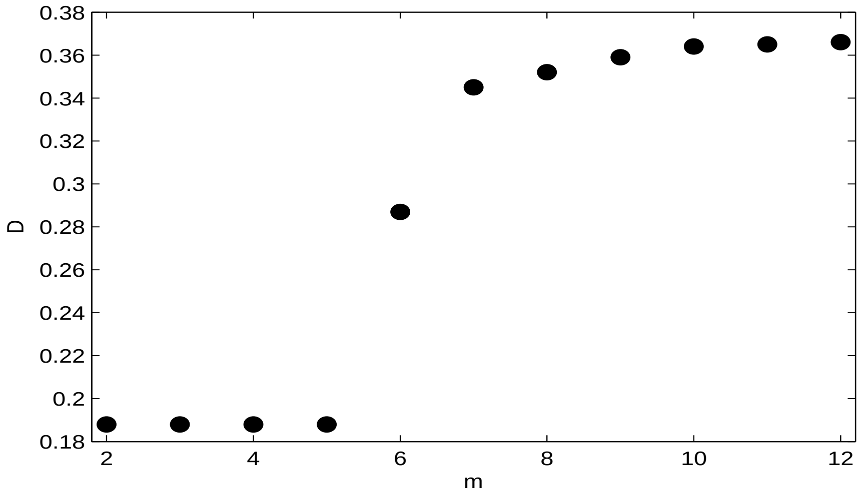

Fig. 5. Absolute difference between conditional probabilities $P_c(1|u_m)$ and $P_i(1|u_m)$ as a function of m for 117 individual 2-strategy agents plus two couples of probabilistic agents with one similar strategy. The phase transition is observed for $m = 6$. Points are averages over 100 outcomes of 11×11 periodic lattices after 50,000 time steps.

clusters increases, in correspondence with the results presented in [3]. Although we have centered on a particular model, the described behavior dynamics and the conclusions can be generalized to similar models with interaction promoting concerted behavior among agents. We believe that this approach might allow a simplified theoretical study of certain local interactions by using the framework developed for the standard MG.

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