Deformation-quantization modules on complex Poisson manifolds

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A DQ-algebra on a complex manifold X is a $\mathbb{C}[[\hbar]]$-algebra locally isomorphic to an algebra $((\mathcal{O}_X[[\hbar]], \star)$ where the product $\star$ is associative and given by bidifferential operators. More generally, a DQ-algebroid is a $\mathbb{C}[[\hbar]]$-linear algebroid stack locally equivalent to a DQ-algebra. The data of a DQ-algebroid on X endows this manifold with a Poisson structure.

We generalize to this framework several notions and results both of complex analytic geometry and of $\mathcal{D}$-module theory. In particular, we construct the dualizing complex for DQ-modules and prove a relative finiteness and duality theorem for such modules.

Then we construct the Hochschild homology of a DQ-algebroid, the Hochschild class of coherent DQ-modules and prove that such classes are compatible to composition of kernels. When the star product is commutative, one recovers the Chern class. On the other hand, when the Poisson structure is symplectic, one recovers the Euler class of $\mathcal{D}$-modules.