Binomial D-modules

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Binomial D-modules are quotients of the Weyl algebra by left ideals whose generators consist of an arbitrary Z^d-graded binomial ideal I in the commutative polynomial ring of partial derivatives with constant complex coefficients, along with Euler operators defined by the grading and a complex parameter vector c. The study of these D-modules, which generalize classical Horn hypergeometric systems, is based on the approach by Gel'fand, Kapranov and Zelevinsky.

We determine the parameters c for which a binomial D-module (i) is (regular) holonomic, (ii) decomposes as a direct sum indexed by the primary components of I; and (iii) has holonomic rank greater than the rank for generic c. In each of these three cases, the parameters in question are precisely those outside of a certain explicitly described affine subspace arrangement. In the special case of Horn hypergeometric D-modules, when I is a lattice basis ideal, we furthermore compute the generic holonomic rank combinatorially and write down a basis of solutions in terms of associated A-hypergeometric functions. The main tools are an explicit lattice point description of the primary components of an arbitrary binomial ideal in characteristic zero, together with the Euler-Koszul complexes developed by Matusevich, Miller and Walther.

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