A Poincare-Bendixson theorem for homogeneous vector fields

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In one complex dimension, a holomorphic germ tangent to the identity $f(z) = z + a_k z^k + o(z^k)$ is locally topologically conjugated to the time-1 map of the homogeneous vector field $z^k \partial/\partial z$. In particular, the study of the real flow of (complex) homogeneous vector fields in (complex) dimension one provides a large amount of informations on the local dynamics of functions tangent to the identity.

This suggests that the study of the real flow of complex homogeneous vector fields might also help to understand the local dynamics of holomorphic map tangent to the identity in complex dimension two, at least in generic cases. In this talk I shall describe how (using ideas introduced by F. Bracci, F. Tovena and myself) it is possible to reduce the study of the real 1-dimensional flow of a complex 2-dimensional homogenous vector field to the study of the geodesic flow of a meromorphic connnection on the complex projective line, and how to use this reduction to prove a Poincare-Bendixson theorem completely describing the recurrent behavior of such flows. (Joint work with F. Tovena)