From $\mathcal{D}$-modules to deformation quantization modules

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Denote by $\mathbf{k}_0$ the ring $\mathbb{C}[[\hbar]]$ and by X a complex manifold. We shall show how the study of $\mathcal{D}$-modules on X, that is, modules over the ring $\mathcal{D}_X$ of holomorphic differential operators, naturally leads to the notion of DQ-modules (DQ: deformation quantization). A DQ-module on X is a module over an algebra (more generally, an algebroid stack) locally isomorphic to an algebra $((\mathcal{O}_X[[\hbar]], \star)$ where the product $\star$ is an associative and $\mathbf{k}_0$-bilinear map given by bi-differential operators. Note that this product defines a Poisson structure on X.

In this course, we start by recalling the main constructions and properties of $\mathcal{D}$-modules, then we briefly discuss algebroid stacks and DQ-modules.

We will freely use the language of sheaves and derived categories. For a short introduction to derived categories and sheaves, see [2, Chapters 1,2], for a detailled exposition, see [3] or the original [4]. For $\mathcal{D}$-modules, see [1].

[1] M. Kashiwara *$\mathcal{D}$-modules and Microlocal Calculus,* Translations of Mathematical Monographs, **217** American Math. Soc. (2003).

[2] M. Kashiwara and P. Schapira, *Sheaves on Manifolds,* Grundlehren der Math. Wiss. **292** Springer-Verlag (1990).

[3] ———, *Categories and Sheaves,* Grundlehren der Math. Wiss. **332** Springer-Verlag (2005).

[4] S-G-A 4, Sém. Géom. Algébrique (1963-64) by M. Artin, A. Grothendieck and J-L. Verdier, *Théorie des topos et cohomologie étale des schémas,* Lecture Notes in Math. **269, 270, 305**, Springer-Verlag (1972/73).