INTRODUCTION

ALICIA DICKENSTEIN*, FRANK-OLAF SCHREYER†, AND ANDREW J. SOMMESE‡

In the last decade, there has been a burgeoning of activity in the design and implementation of algorithms for algebraic geometric computation. Some of these algorithms were originally designed for abstract algebraic geometry, but now are of interest for use in applications and some of these algorithms were originally designed for applications, but now are of interest for use in abstract algebraic geometry.

The Workshop on *Algorithms in Algebraic Geometry* that was held in the framework of the IMA Annual Program Year in Applications of Algebraic Geometry by the Institute for Mathematics and Its Applications on September 18–22, 2006 at the University of Minnesota is one tangible indication of the interest. One hundred ten participants from eleven countries and twenty states came to listen to the many talks; discuss mathematics; and pursue collaborative work on the many faceted problems and the algorithms, both symbolic and numeric, that illuminate them.

This volume of articles captures some of the spirit of the IMA Workshop.

Daniel Bates, Chris Peterson, and Andrew Sommese show how the numerical algebraic geometry calculations originally aimed at applications may be used to quickly compute information about joins of varieties.

Frederic Bihan, J. Maurice Rojas, and Frank Sottile show the existence of fewnomial systems of polynomials whose number of positive real solutions equals a theoretical upper bound.

Sara Billey and Ravi Vakil, blend combinatorics and geometry, to give an algorithm for algebraically finding all flags in any zero-dimensional intersection of Schubert varieties with respect to any number of flags. This leads to a very easy method of checking that a structure constant for flag manifolds is zero.

Antonio Cafure, Guillermo Matera, and Ariel Waissbein study the problem of finding the inverse image of a point in the image of a rational map between vector spaces over finite fields. This problem is of great current interest in the coding community because it lies at the heart of a new approach to public key encryption in a world where it is ever more likely that quantum computers will allow quick factorization of integers and thereby dissolve the current encryption methods underlying secure transactions, e.g., by financial institutions.

*Departamento de Matemática, FCEN, Universidad de Buenos Aires, Argentina
†Mathematik und Informatik, Universität des Saarlandes, Germany
‡Department of Mathematics, University of Notre Dame, Notre Dame, IN 46556-4618, U.S.A.

Anton Leykin, Jan Verschelde, and Ailing Zhao study different approaches to the multiplicity structure of a singular isolated solution of a polynomial system, and, by so doing, present a "deflation method" based on higher order derivatives which effectively restores the quadratic convergence of Newton's method lost due to the singularity of the isolated solution.

Heidi Mork and Ragni Piene study polar and reciprocal varieties for possibly singular real algebraic curves. They show that in the case when only ordinary singularities are present on the curve, these associated varieties contain nonsingular points on all components of the original curve and can be used to investigate the components of the curve. They also give an example of a curve with nonordinary singularities for which this fails.

Jiawang Nie, Pablo Parrilo, and Bernd Sturmfels investigate the k-ellipse. This is the plane algebraic curve consisting of all points such that the sum of distances from k given points is a fixed number. They show how to write its defining polynomial equation as the determinant of a symmetric matrix of linear polynomials. Their representation extends to arbitrary dimensions, and it leads to new geometric applications of semidefinite programming.

Andrew Sommese, Jan Verschelde, and Charles Wampler present a numerical method (based on their diagonal homotopy algorithm for intersecting varieties) with the potential to allow solution of polynomial systems, which have relatively few solutions, but whose Bézout numbers are too large to allow solution by the usual homotopy continuation methods.

Other interesting subjects presented at the Workshop include the computation of the intersection and self-intersection loci of parameterized space algebraic surfaces and the study of projections methods for the topology of algebraic curves and surfaces, which are of interest in Computer Aided Geometric Design, algorithms for finding generators of rings of invariants of algebraic groups, the relation between the structure of Gröbner bases and the decomposition of polynomial systems and the complexity of Gröbner basis computations for regular and semi-regular systems, the factorization of sparse polynomials over number fields, counting rational points on varieties over finite fields, algorithms for mixed volume computation, the effectiveness of number theory in algebraic geometry, applications of monomial ideals and computational algebra in the reverse engineering of biological networks, the description of Newton polytopes of implicit equations using Tropical Geometry, the mathematical description of Maple's algebraic curves package and algorithms for finding all real solutions contained in a complex algebraic curve. The interested reader can consult the talk materials and online videos of the lectures at `www.ima.umn.edu/2006-2007/W9.18-22.06/abstracts.html`.