Práctica 3
Métodos algebraicos ...

Las numeraciones corresponden a las de los ejercicios del Capítulo 2: "Laplacian dynamics, complex and detailed balancing del borrador del libro "Algebraic Methods for Biochemical Reaction Networks. Este debería ser el Capítulo 3, pero los números se corrieron en distintas compilaciones, lo siento. Una corrección importante es que en el enunciado del Theorem 2.1.9, en la última línea, donde dice ρ_i^G debe decir ρ_j^G.

1. Exercise 2.2.
2. Exercise 2.6.
3. Exercise 2.7.
4. Exercise 2.10.
5. Consider the following MAK CRN reaction network:

$$
\begin{aligned}
&S_0 + E \underset{k_2}{\overset{k_1}{\rightleftarrows}} ES_0 \underset{k_4}{\overset{k_3}{\rightleftarrows}} S_1 + E \\
&S_1 + F \underset{\ell_2}{\overset{\ell_1}{\rightleftarrows}} FS_1 \underset{\ell_4}{\overset{\ell_3}{\rightleftarrows}} S_0 + F \ .
\end{aligned} \tag{1}
$$

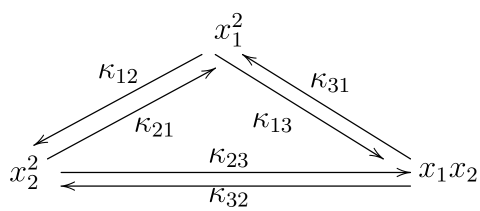
and the following reaction network, in which the substrate S_1 in the first set of reactions acts as an enzyme in the second set of reactions:

$$S_0 + E \underset{k_2}{\overset{k_1}{\rightleftarrows}} ES_0 \underset{k_4}{\overset{k_3}{\rightleftarrows}} S_1 + E \qquad (2)$$

$$T_0 + S_1 \underset{\ell_2}{\overset{\ell_1}{\rightleftarrows}} T_0S_1 \underset{\ell_4}{\overset{\ell_3}{\rightleftarrows}} T_1 + S_1 \ .$$

In both cases, compute the deficiency of the network and describe the varieties (in the positive orthant of rate constant space) of those vectors of rate constants for which (a) the system is detailed balanced, (b) the system is complex balanced.

6. Let $s = 2$, $m = 3$, and consider be the following network:



(i) Write down the resulting mass-action kinetics system

$$\dot{x} = (f_1, (x), f_2(x)).$$

(ii) Describe all rate constants for which the steady state ideal $I = \langle f_1, f_2 \rangle$ is binomial.

(iii) Describe all rate constants for which the system is *detailed balanced.*

(iv) Set $k_{31} = k_{23} = 2, k_{13} = k_{32} = k_{12} = 1, k_{21} = 4$, and check that the system is detailed balanced in this case but I is not binomial. Find its "positive real radical" $J := \{g \in \mathbb{R}[x_1, x_2] \ : \ g(c) = 0 \text{ for all } c \in V_{\mathbb{R}_{>0}}(I)\}$. Is J binomial?

(v) Describe all rate constants for which the system is *complex balanced.* If possible, find rate constants for which the system is complex balanced but not detailed balanced.

7. Consider the reactions

$$A + B \xrightarrow{k_1} C \qquad B \xrightarrow{k_2} A + C \qquad C \xrightarrow{k_3} B.$$

Let x_A, x_B, x_C denote the concentration of species A, B, C respectively.

(a) Find the ODE system associated to the reactions using the mass-action assumption.

(b) Show that $x_B + x_C$ is conserved.

(c) Show that a positive steady state always exist and is unique.

(d) Show that the value of x_A is constant at a positive steady state independently of the system's initial conditions (i.e., absolute concentration robustness exists for x_A).

(e) Is this true for all nonnegative steady states?