

restart : with(LinearAlgebra) : with(convex) : with(combinat) : with(Optimization) :  
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#set n

n := 5 :

#since the tables are going to be big, increase the maximum allowed size for tables

interface(rtablesize=4 + 2·n) :

#set T[-1] and the matrices A, C and Csimple

T[-1] := 1 :

A := Transpose(Matrix([[1, 0, 0], [0, 1, 0], [0, 0, 1], seq([1, i, -i], i=1..n), seq([1, i, 1-i], i=1..n), [0, 0, 0]]));

C := Transpose(Matrix([[1, 0, 0], [0, 1, 0], [0, 0, 1], seq([T[i], 0, 0], i=0..n-1), seq([K[i]·T[i]-1] + L[i]·T[i], K[i]·T[i-1], L[i]·T[i]], i=0..n-1), [-S, -E, -F]])) :

Csimple := Transpose(Matrix([[1, 0, 0], [0, 1, 0], [0, 0, 1], seq([1, 0, 0], i=0..n-1), seq([1, M[i], 1-M[i]], i=0..n-1), [-S, -E, -F]]))

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 1 & 2 & 3 & 4 & 5 & 0 \\ 0 & 0 & 1 & -1 & -2 & -3 & -4 & -5 & 0 & -1 & -2 & -3 & -4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -S \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & M_0 & M_1 & M_2 & M_3 & M_4 & -E \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1-M_0 & 1-M_1 & 1-M_2 & 1-M_3 & 1-M_4 & -F \end{bmatrix} \quad (1)$$

# here we define the procedure Foundoriginaltriang that we will use in the end of the script.

# this will be use when we need to recover the triangulation in L1 that gave a element in L7 used to obtaing a region of multistationarity

Foundoriginaltriang := proc(original, T)

local k, aux;

aux := T;

for k from 1 to numelems(original) do

if {op(T)} subset {op(original[k][2])} then aux := original[k][1] fi

od:

aux;

end proc:

# here we define validpolytopesindex as set of triples that index all zero 3x3 minors of Csimple

# this will be used to pass from L1 to L2

validpolytopesindex := [ ]:

for i1 from 1 to ColumnDimension(Csimple) - 1 do

for i2 from i1 + 1 to ColumnDimension(Csimple) - 1 do

for i3 from i2 + 1 to ColumnDimension(Csimple) - 1 do

if Determinant(Csimple[1..3, [i1, i2, i3]]) ≠ 0 then

Determinant(Csimple[1..3, [i1, i2, i3]]);

validpolytopesindex := [op(validpolytopesindex), [i1, i2, i3]];

end if

end do end do end do:

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# here we import L1 from a file output from SAGE

L1 := parse(ImportData( ) ) :
# here we do step (2) of Algorithm to obtain L2 from L1

L2 := { } :
originals := { } :
for i from 1 to numelems(L1) do
  auxi2 := [ ];
  for l from 1 to numelems(L1[i]) do
    auxi := [0, 0, 0, 0];
    for j from 1 to 4 do
      # this line is needed because on SAGE the vertex are indexed beginning with 0 and we want that they
      # start from 1
      auxi[j] := L1[i][l][j] + 1;
    od :
    # the next "if" makes Step (2) passing from L1 to L2 only the simplexes with the last vertex
    if auxi[4] = ColumnDimension(C) then auxi2 := [op(auxi2), auxi]; fi:
    od:
    if not(member(auxi2, L2)) then
      L2 := {op(L2), auxi2};
      # the originals list is a link between L1 and L2 used after to recover elements of L1 from L2
      originals := originals union { [L1[i], auxi2] } :
      fi:
    od:
  # here we do step (3) of Algorithm to obtain L3 from L2

  L3 := { } :
  originals2 := { } :
  for i from 1 to numelems(L2) do
    auxi := [ ];
    for l from 1 to numelems(L2[i]) do

      # the next "if" makes Step (3) passing from L2 to L3 only simplexes whose corresponding minor is
      # not zero
      if { [L2[i][l][1], L2[i][l][2], L2[i][l][3]] } subset {op(validpolytopesindex)} then
        auxi := [op(auxi), L2[i][l]];
        fi:
      od:
      if not(member(auxi, L3)) then
        L3 := L3 union {auxi};
        # the originals2 list is a link between L2 and L3 used after to recover elements of L2 from L3
        originals2 := originals2 union { [L2[i], auxi] } :
        fi:
      od:
    # here we do step (4) of Algorithm to obtain L4 from L3

    originals3 := { } :
    L4 := { } :
    for i from 1 to numelems(L3) do
      auxi := [ ];

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for  $l$  from 1 to numelems( $L3[i]$ ) do
   $auxi2 := L3[i][l]$ ;
for  $j$  from 4 to  $n + 3$  do
   $bb := member(j, L3[i][l], 'pp')$ ;
  if  $bb = true$  then
     $auxi2[pp] := 1$ ; fi; od;
   $auxi := sort([op(auxi), sort(auxi2)])$ ;
  od;
  if not( $member(auxi, L4)$ ) then
     $L4 := L4 \cup \{auxi\}$ ;
    # the originals3 list is a link between L3 and L4 used after to recover elements of L3 from L4
     $originals3 := originals3 \cup \{[L3[i], auxi]\}$ ;
  fi;
od;
# here we do step (5) of Algorithm to obtain L5 from L4

 $L5 := \{ \}$ ;
for  $i$  from 1 to numelems( $L4$ ) do
   $auxi := 0$ ;
  for  $j$  from  $i + 1$  to numelems( $L4$ ) while  $auxi = 0$  do
  if numelems( $\{op(L4[i])\} \cap \{op(L4[j])\}$ ) = numelems( $L4[i]$ ) then
     $auxi := 1$ ;
  fi;
  od;
  if  $auxi = 0$  then
     $L5 := L5 \cup \{L4[i]\}$ 
  fi;
od;
# the following is just a information check

print("This list L1 is the whole list.");
print("This list L2 consider only the simpleces having the origin.");
print("This list L3 takes out the simpleces corresponding to zero determinant.");
print("This list L4 replaces 4,5,...,n+3 by 1.");
  print("This list L5 takes out the triangulations T such that there is another triangulation T'
  containing T.");
print("Number of elements of L1, L2,L3, L4, and L5. And the matrix Csimple");
nops(L1); nops(L2); nops(L3); nops(L4); nops(L5); Csimple;
print( );

      "This list L1 is the whole list."
      "This list L2 consider only the simpleces having the origin."
      "This list L3 takes out the simpleces corresponding to zero determinant."
      "This list L4 replaces 4,5,...,n+3 by 1."
      "This list L5 takes out the triangulations T such that there is another triangulation T' containing
      T."
      "Number of elements of L1, L2,L3, L4, and L5. And the matrix Csimple"

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122835

28044

4560

177

149

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -S \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & M_0 & M_1 & M_2 & M_3 & M_4 & -E \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 - M_0 & 1 - M_1 & 1 - M_2 & 1 - M_3 & 1 - M_4 & -F \end{bmatrix}$$

(2)

*#the following counts and displays how many elements of L5 has a determined size  
 #this can be used to guess what will be a good candidate for k*

```
count2 := [seq(0, i = 1 ..nops(L5[nops(L5)]) )]:
for i from 1 to numelems(L5) do
count := nops(L5[i]) :
count2[count] := count2[count] + 1 :
od:
for i from 1 to nops(count2) do
print(there is, count2[i], configurations with, i, valid polytopes);
od;
for J in L5 do
print(J);
od:

    there is, 0, configurations with, 1, valid polytopes
    there is, 0, configurations with, 2, valid polytopes
    there is, 8, configurations with, 3, valid polytopes
    there is, 0, configurations with, 4, valid polytopes
    there is, 37, configurations with, 5, valid polytopes
    there is, 0, configurations with, 6, valid polytopes
    there is, 65, configurations with, 7, valid polytopes
    there is, 0, configurations with, 8, valid polytopes
    there is, 34, configurations with, 9, valid polytopes
    there is, 0, configurations with, 10, valid polytopes
    there is, 5, configurations with, 11, valid polytopes
        [[1, 2, 3, 14], [1, 2, 10, 14], [1, 2, 10, 14]]
        [[1, 2, 3, 14], [1, 2, 11, 14], [1, 2, 11, 14]]
        [[1, 2, 3, 14], [1, 2, 12, 14], [1, 2, 12, 14]]
        [[1, 2, 3, 14], [1, 2, 13, 14], [1, 2, 13, 14]]
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        [[1, 2, 3, 14], [1, 3, 12, 14], [1, 3, 12, 14]]
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[[1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 10, 14], [1, 10, 13, 14], [2, 3, 9, 14], [2, 9, 10, 14], [2, 10, 13, 14]]  
[[1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 10, 14], [1, 10, 13, 14], [2, 3, 10, 14], [2, 10, 13, 14], [3, 9, 10, 14]]  
[[1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 10, 14], [1, 10, 13, 14], [2, 3, 13, 14], [3, 9, 10, 14], [3, 10, 13, 14]]  
[[1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 11, 14], [1, 11, 13, 14], [2, 3, 9, 14], [2, 9, 11, 14], [2, 11, 13, 14]]  
[[1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 11, 14], [1, 11, 13, 14], [2, 3, 11, 14], [2, 11, 13, 14], [3, 9, 11, 14]]  
[[1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 11, 14], [1, 11, 13, 14], [2, 3, 13, 14], [3, 9, 11, 14], [3, 11, 13, 14]]  
[[1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 12, 14], [1, 12, 13, 14], [2, 3, 9, 14], [2, 9, 12, 14], [2, 12, 13, 14]]  
[[1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 12, 14], [1, 12, 13, 14], [2, 3, 12, 14], [2, 12, 13, 14], [3, 9, 12, 14]]  
[[1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 12, 14], [1, 12, 13, 14], [2, 3, 13, 14], [3, 9, 12, 14], [3, 12, 13, 14]]  
[[1, 2, 13, 14], [1, 3, 10, 14], [1, 3, 10, 14], [1, 3, 12, 14], [1, 12, 13, 14], [2, 3, 12, 14], [2, 12, 13, 14]]  
[[1, 2, 13, 14], [1, 3, 10, 14], [1, 3, 10, 14], [1, 3, 12, 14], [1, 12, 13, 14], [2, 3, 13, 14], [3, 12, 13, 14]]  
[[1, 2, 13, 14], [1, 3, 10, 14], [1, 3, 11, 14], [1, 3, 13, 14], [1, 10, 11, 14], [2, 3, 13, 14], [3, 10, 11, 14]]  
[[1, 2, 13, 14], [1, 3, 10, 14], [1, 10, 11, 14], [1, 11, 13, 14], [2, 3, 10, 14], [2, 10, 11, 14], [2, 11, 13, 14]]  
[[1, 2, 13, 14], [1, 3, 10, 14], [1, 10, 11, 14], [1, 11, 13, 14], [2, 3, 11, 14], [2, 11, 13, 14], [3, 10, 11, 14]]

10, 11, 14]]  
[[1, 2, 13, 14], [1, 3, 10, 14], [1, 10, 11, 14], [1, 11, 13, 14], [2, 3, 13, 14], [3, 10, 11, 14], [3, 11, 13, 14]]  
[[1, 2, 13, 14], [1, 3, 10, 14], [1, 10, 12, 14], [1, 12, 13, 14], [2, 3, 10, 14], [2, 10, 12, 14], [2, 12, 13, 14]]  
[[1, 2, 13, 14], [1, 3, 10, 14], [1, 10, 12, 14], [1, 12, 13, 14], [2, 3, 12, 14], [2, 12, 13, 14], [3, 10, 12, 14]]  
[[1, 2, 13, 14], [1, 3, 10, 14], [1, 10, 12, 14], [1, 12, 13, 14], [2, 3, 13, 14], [3, 10, 12, 14], [3, 12, 13, 14]]  
[[1, 2, 3, 14], [1, 2, 10, 14], [1, 2, 13, 14], [1, 10, 11, 14], [1, 11, 12, 14], [1, 12, 13, 14], [2, 10, 11, 14], [2, 11, 12, 14], [2, 12, 13, 14]]  
[[1, 2, 3, 14], [1, 3, 9, 14], [1, 3, 12, 14], [1, 9, 10, 14], [1, 10, 11, 14], [1, 11, 12, 14], [3, 9, 10, 14], [3, 10, 11, 14], [3, 11, 12, 14]]  
[[1, 2, 9, 14], [1, 2, 11, 14], [1, 2, 13, 14], [1, 3, 9, 14], [1, 11, 12, 14], [1, 12, 13, 14], [2, 3, 9, 14], [2, 11, 12, 14], [2, 12, 13, 14]]  
[[1, 2, 10, 14], [1, 2, 12, 14], [1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 10, 14], [1, 12, 13, 14], [2, 3, 9, 14], [2, 9, 10, 14], [2, 12, 13, 14]]  
[[1, 2, 10, 14], [1, 2, 12, 14], [1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 10, 14], [1, 12, 13, 14], [2, 3, 10, 14], [2, 12, 13, 14], [3, 9, 10, 14]]  
[[1, 2, 11, 14], [1, 2, 13, 14], [1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 10, 14], [1, 10, 11, 14], [2, 3, 9, 14], [2, 9, 10, 14], [2, 10, 11, 14]]  
[[1, 2, 11, 14], [1, 2, 13, 14], [1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 10, 14], [1, 10, 11, 14], [2, 3, 10, 14], [2, 10, 11, 14], [3, 9, 10, 14]]  
[[1, 2, 11, 14], [1, 2, 13, 14], [1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 10, 14], [1, 10, 11, 14], [2, 3, 11, 14], [3, 9, 10, 14], [3, 10, 11, 14]]  
[[1, 2, 12, 14], [1, 3, 9, 14], [1, 9, 10, 14], [1, 10, 11, 14], [1, 11, 12, 14], [2, 3, 9, 14], [2, 9, 10, 14], [2, 10, 11, 14], [2, 11, 12, 14]]  
[[1, 2, 12, 14], [1, 3, 9, 14], [1, 9, 10, 14], [1, 10, 11, 14], [1, 11, 12, 14], [2, 3, 10, 14], [2, 10, 11, 14], [2, 11, 12, 14], [3, 9, 10, 14]]  
[[1, 2, 12, 14], [1, 3, 9, 14], [1, 9, 10, 14], [1, 10, 11, 14], [1, 11, 12, 14], [2, 3, 11, 14], [2, 11, 12, 14], [3, 9, 10, 14], [3, 10, 11, 14]]  
[[1, 2, 12, 14], [1, 3, 9, 14], [1, 9, 10, 14], [1, 10, 11, 14], [1, 11, 12, 14], [2, 3, 12, 14], [3, 9, 10, 14], [3, 10, 11, 14], [3, 11, 12, 14]]  
[[1, 2, 13, 14], [1, 3, 9, 14], [1, 3, 9, 14], [1, 3, 11, 14], [1, 11, 12, 14], [1, 12, 13, 14], [2, 3, 11, 14], [2, 11, 12, 14], [2, 12, 13, 14]]  
[[1, 2, 13, 14], [1, 3, 9, 14], [1, 3, 9, 14], [1, 3, 11, 14], [1, 11, 12, 14], [1, 12, 13, 14], [2, 3, 12, 14], [2, 12, 13, 14], [3, 11, 12, 14]]  
[[1, 2, 13, 14], [1, 3, 9, 14], [1, 3, 9, 14], [1, 3, 11, 14], [1, 11, 12, 14], [1, 12, 13, 14], [2, 3, 13, 14], [3, 11, 12, 14], [3, 12, 13, 14]]  
[[1, 2, 13, 14], [1, 3, 9, 14], [1, 3, 10, 14], [1, 3, 12, 14], [1, 9, 10, 14], [1, 12, 13, 14], [2, 3,



3, 10, 14], [2, 10, 11, 14], [2, 11, 12, 14], [2, 12, 13, 14], [3, 9, 10, 14]]  
 [[1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 10, 14], [1, 10, 11, 14], [1, 11, 12, 14], [1, 12, 13, 14], [2,  
 3, 11, 14], [2, 11, 12, 14], [2, 12, 13, 14], [3, 9, 10, 14], [3, 10, 11, 14]]  
 [[1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 10, 14], [1, 10, 11, 14], [1, 11, 12, 14], [1, 12, 13, 14], [2,  
 3, 12, 14], [2, 12, 13, 14], [3, 9, 10, 14], [3, 10, 11, 14], [3, 11, 12, 14]]  
 [[1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 10, 14], [1, 10, 11, 14], [1, 11, 12, 14], [1, 12, 13, 14], [2,  
 3, 13, 14], [3, 9, 10, 14], [3, 10, 11, 14], [3, 11, 12, 14], [3, 12, 13, 14]]

(3)

# in the following we check for each element of L5 the conditions are needed for it to be positively decorated by Csimple

# the corresponding subsets S of the T in L found are stored in "solutions"

# finally in the variable "allsolutions" we keep the candidates to give at least  $k=2 \cdot \text{floor}\left(\frac{n}{2}\right)$

+ 1 regions

allsolutions := { }:

for J in L5 do

if numelems(J)  $\geq 2 \cdot \text{floor}\left(\frac{n}{2}\right) + 1$  then

Jused := [ ] : solutions := { [Jused, {1, E, F, S, seq(1 - M[i], i=0 ..n - 1)}] } : solutionsaux := { }:

for j in J do

solutionsaux := { }:

for l in solutions do

for i from 1 to 4 do det[i] := Determinant(Csimple[1 ..3, subsop(i=NULL, j)]) : od:

Jused := l[1]; conditions := l[2];

conditionsnewa := { -det[1], det[2], -det[3], det[4]};

conditionsnewb := { det[1], -det[2], det[3], -det[4]};

if evalb(numelems(conditions intersect conditionsnewa)  $\geq 1$  and numelems(conditions intersect conditionsnewb)  $\geq 1$ ) = true then

solutionsaux := solutionsaux union { [Jused, conditions]};

fi:

if evalb(numelems(conditions intersect conditionsnewb) = 0) = true then

solutionsaux := solutionsaux union { [op(Jused), j], conditions union conditionsnewa };

fi:

if evalb(numelems(conditions intersect conditionsnewa) = 0) = true then

solutionsaux := solutionsaux union { [op(Jused), j], conditions union conditionsnewb };

fi:

od:

solutions := solutionsaux;

od:

for k in solutions do

if numelems(k[1])  $\geq 2 \cdot \text{floor}\left(\frac{n}{2}\right) + 1$  then allsolutions := allsolutions union {k}; fi:

od:

fi:

od:

```

print("Number of solutions to try", numelems(allsolutions));
      "Number of solutions to try", 77
# in this part we obtain L7 from "allsolutions"
# we do this searching in "allsolutions" for the elements for which there are viable parameters
  satistaying the conditions
# this is the only numerical part of the whole script
# in the end each J in L7 will contain:
#J[1] = list of simplexes;
# J[2]= corresponding conditions;
# J[3]= a list of real numbers which are viable values for the parameters

interface(displayprecision = 6) : L7 := { } :
for j in allsolutions do
  conditions := j[2] :
  Jused := j[1] :
try
  Min := Minimize  $\left( 1, \left\{ \text{seq} \left( \text{conditions}[j] \geq \frac{1}{10000}, j = 1 \dots \text{numelems}(\text{conditions}) \right) \right\}, \text{assume} \right.$ 
    = nonnegative, iterationlimit = 100 ) :
  #print("solution is", op(Min[2]));
  #print("In this case there is", numelems(Jused), "positive solutions");
  L7 := L7 union { [j[1], j[2], Min[2]] } :
catch:
  #print(Nao foi encontrada solucao);
end try:
end do:

solutionsaux := { } :
for k from 1 to numelems(L7) do
  solutionsaux := solutionsaux union { [L7[k][1], L7[k][2] minus {1, E, F, S, seq(1 - M[i], i = 0 ..n
    - 1), seq(M[i], i = 0 ..n - 1)}, L7[k][3]] } :
od:
  L7 := solutionsaux :

  solutionsaux := { } :
for k from 1 to numelems(L7) do
  aux := 0 :
for j from k + 1 to numelems(L7) do
if evalb(L7[k][2] subset L7[j][2]) then
  aux := 1
fi:
od:
if aux = 0 then
  solutionsaux := solutionsaux union { [L7[k][1], L7[k][2], L7[k][3]] } :
fi:
od:
  L7 := solutionsaux :

```

(4)

```

print("there are", numelems(L7), "maximal regions in which there are", 2 * floor( $\frac{n}{2}$ ) + 1,
      "solutions");
print("the original triangulations, simpleces positively decorated, regions, and a point on it are");

```

*# this part recovers the original triangulations from the final sets obtained*

**for** *i* **from** 1 **to** numelems(L7) **do**

*Foundoriginaltriang(originals, Foundoriginaltriang(originals2, Foundoriginaltriang(originals3,*  
*L7[i][1])));*

*L7[i][1];*

*L7[i][2];*

*L7[i][3];*

**od;**

"there are", 23, "maximal regions in which there are", 5, "solutions"

"the original triangulations, simpleces positively decorated, regions, and a point on it are"

$[[0, 1, 2, 8], [0, 1, 2, 13], [0, 1, 8, 9], [0, 1, 9, 13], [0, 5, 10, 13], [0, 9, 10, 13], [1, 5, 10, 11],$   
 $[1, 5, 10, 13], [1, 5, 11, 12], [1, 5, 12, 13], [1, 7, 12, 13], [1, 9, 10, 13], [5, 7, 12, 13]]$   
 $[[1, 2, 3, 14], [1, 2, 10, 14], [1, 2, 11, 14], [1, 2, 13, 14], [1, 2, 13, 14]]$

$\{-EM_1 - FM_1 + E, -EM_2 - FM_2 + E, -EM_4 - FM_4 + E, -SM_1 - F + S, -SM_2 - F + S,$   
 $-SM_4 - F + S\}$

$[E = 1.000000, F = 0.250000, S = 1.000075, M_0 = 0.999900, M_1 = 0.749900, M_2 = 0.749900, M_3$   
 $= 0.999900, M_4 = 0.749900]$

$[[0, 1, 2, 8], [0, 1, 2, 13], [0, 1, 8, 9], [0, 1, 9, 13], [0, 6, 11, 13], [0, 9, 10, 13], [0, 10, 11,$   
 $13], [1, 6, 7, 11], [1, 6, 7, 13], [1, 6, 11, 13], [1, 7, 11, 12], [1, 9, 10, 13], [1, 10, 11, 13]]$   
 $[[1, 2, 3, 14], [1, 2, 10, 14], [1, 2, 12, 14], [1, 10, 11, 14], [1, 11, 12, 14]]$

$\{M_2 - M_1, M_2 - M_3, -EM_1 - FM_1 + E, EM_2 + FM_2 - E, -EM_3 - FM_3 + E, -SM_1 - F + S,$   
 $-SM_3 - F + S, EM_1 - EM_2 + FM_1 - FM_2 - SM_1 + SM_2, -EM_2 + EM_3 - FM_2 + FM_3$   
 $+ SM_2 - SM_3\}$

$[E = 0.399860, F = 0.000197, S = 2.200280, M_0 = 0.999900, M_1 = 0.999258, M_2 = 0.999782, M_3$   
 $= 0.999258, M_4 = 0.999900]$

$[[0, 1, 2, 8], [0, 1, 2, 13], [0, 1, 8, 9], [0, 1, 9, 13], [0, 7, 12, 13], [0, 9, 10, 13], [0, 10, 12,$   
 $13], [1, 7, 12, 13], [1, 9, 10, 13], [1, 10, 12, 13]]$   
 $[[1, 2, 3, 14], [1, 2, 10, 14], [1, 2, 13, 14], [1, 10, 11, 14], [1, 11, 13, 14]]$

$\{M_2 - M_1, M_2 - M_4, -EM_1 - FM_1 + E, EM_2 + FM_2 - E, -EM_4 - FM_4 + E, -SM_1 - F + S,$   
 $-SM_4 - F + S, EM_1 - EM_2 + FM_1 - FM_2 - SM_1 + SM_2, -EM_2 + EM_4 - FM_2 + FM_4$   
 $+ SM_2 - SM_4\}$

$[E = 0.399860, F = 0.000197, S = 2.200280, M_0 = 0.999900, M_1 = 0.999258, M_2 = 0.999782, M_3$   
 $= 0.999900, M_4 = 0.999258]$

[[0, 1, 2, 8], [0, 1, 2, 13], [0, 1, 8, 9], [0, 1, 9, 13], [0, 7, 12, 13], [0, 9, 11, 13], [0, 11, 12, 13], [1, 7, 12, 13], [1, 9, 11, 13], [1, 11, 12, 13]]

[[1, 2, 3, 14], [1, 2, 10, 14], [1, 2, 13, 14], [1, 10, 12, 14], [1, 12, 13, 14]]

$\{M_3 - M_1, M_3 - M_4, -EM_1 - FM_1 + E, EM_3 + FM_3 - E, -EM_4 - FM_4 + E, -SM_1 - F + S, -SM_4 - F + S, EM_1 - EM_3 + FM_1 - FM_3 - SM_1 + SM_3, -EM_3 + EM_4 - FM_3 + FM_4 + SM_3 - SM_4\}$

[ $E=0.399860, F=0.000197, S=2.200280, M_0=0.999900, M_1=0.999258, M_2=0.999900, M_3=0.999782, M_4=0.999258$ ]

[[0, 2, 8, 13], [0, 4, 8, 13], [1, 2, 7, 11], [1, 2, 7, 13], [1, 7, 11, 12], [2, 4, 8, 10], [2, 4, 8, 13], [2, 4, 10, 13], [2, 7, 11, 13], [2, 10, 11, 13], [4, 7, 11, 13], [4, 10, 11, 13]]

[[1, 2, 3, 14], [1, 3, 9, 14], [1, 3, 9, 14], [1, 3, 11, 14], [1, 3, 12, 14]]

$\{SM_0 - E, SM_2 - E, SM_3 - E, EM_0 + FM_0 - E, EM_2 + FM_2 - E, EM_3 + FM_3 - E\}$

[ $E=0.999900, F=1.000000, S=1.000100, M_0=0.999900, M_1=0.999900, M_2=0.999900, M_3=0.999900, M_4=0.999900$ ]

[[0, 2, 8, 13], [0, 7, 11, 13], [0, 8, 9, 13], [0, 9, 11, 13], [1, 2, 7, 11], [1, 2, 7, 13], [1, 7, 11, 12], [2, 7, 11, 13], [2, 8, 9, 13], [2, 9, 11, 13]]

[[1, 2, 3, 14], [1, 3, 9, 14], [1, 3, 12, 14], [1, 9, 10, 14], [1, 10, 12, 14]]

$\{M_0 - M_1, M_3 - M_1, SM_0 - E, SM_3 - E, EM_0 + FM_0 - E, -EM_1 - FM_1 + E, EM_3 + FM_3 - E, -EM_0 + EM_1 - FM_0 + FM_1 + SM_0 - SM_1, EM_1 - EM_3 + FM_1 - FM_3 - SM_1 + SM_3\}$

[ $E=0.449981, F=0.082246, S=2.301131, M_0=0.917170, M_1=0.845280, M_2=0.999900, M_3=0.910353, M_4=0.999900$ ]

[[0, 2, 8, 13], [0, 7, 11, 13], [0, 8, 10, 13], [0, 10, 11, 13], [1, 2, 7, 11], [1, 2, 7, 13], [1, 7, 11, 12], [2, 7, 11, 13], [2, 8, 10, 13], [2, 10, 11, 13]]

[[1, 2, 3, 14], [1, 3, 9, 14], [1, 3, 12, 14], [1, 9, 11, 14], [1, 11, 12, 14]]

$\{M_0 - M_2, M_3 - M_2, SM_0 - E, SM_3 - E, EM_0 + FM_0 - E, -EM_2 - FM_2 + E, EM_3 + FM_3 - E, -EM_0 + EM_2 - FM_0 + FM_2 + SM_0 - SM_2, EM_2 - EM_3 + FM_2 - FM_3 - SM_2 + SM_3\}$

[ $E=0.449981, F=0.082246, S=2.301131, M_0=0.917170, M_1=0.999900, M_2=0.845280, M_3=0.910353, M_4=0.999900$ ]

[[0, 2, 8, 9], [0, 2, 9, 13], [0, 7, 11, 13], [0, 9, 10, 13], [0, 10, 11, 13], [1, 2, 7, 11], [1, 2, 7, 13], [1, 7, 11, 12], [2, 7, 11, 13], [2, 9, 10, 13], [2, 10, 11, 13]]

[[1, 2, 3, 14], [1, 3, 10, 14], [1, 3, 12, 14], [1, 10, 11, 14], [1, 11, 12, 14]]

$\{M_1 - M_2, M_3 - M_2, SM_1 - E, SM_3 - E, EM_1 + FM_1 - E, -EM_2 - FM_2 + E, EM_3 + FM_3 - E, -EM_1 + EM_2 - FM_1 + FM_2 + SM_1 - SM_2, EM_2 - EM_3 + FM_2 - FM_3 - SM_2\}$

$$+ S M_3 \}$$

$$[E = 0.449981, F = 0.082246, S = 2.301131, M_0 = 0.999900, M_1 = 0.917170, M_2 = 0.845280, M_3 = 0.910353, M_4 = 0.999900]$$

$$[[0, 2, 8, 13], [0, 3, 8, 13], [1, 2, 8, 13], [1, 3, 8, 9], [1, 3, 8, 13], [1, 3, 9, 10], [1, 3, 10, 13], [1, 7, 12, 13], [1, 10, 11, 13], [1, 11, 12, 13], [3, 7, 12, 13], [3, 10, 11, 13], [3, 11, 12, 13]]$$

$$[[1, 2, 9, 14], [1, 2, 11, 14], [1, 2, 13, 14], [1, 11, 12, 14], [1, 12, 13, 14]]$$

$$\{M_3 - M_2, M_3 - M_4, -E M_0 - F M_0 + E, -E M_2 - F M_2 + E, E M_3 + F M_3 - E, -E M_4 - F M_4 + E, -S M_0 - F + S, -S M_2 - F + S, -S M_4 - F + S, E M_2 - E M_3 + F M_2 - F M_3 - S M_2 + S M_3, -E M_3 + E M_4 - F M_3 + F M_4 + S M_3 - S M_4\}$$

$$[E = 0.399860, F = 0.000190, S = 2.200280, M_0 = 0.999275, M_1 = 0.999900, M_2 = 0.999275, M_3 = 0.999791, M_4 = 0.999275]$$

$$[[0, 2, 8, 13], [0, 4, 9, 13], [0, 8, 9, 13], [1, 2, 9, 13], [1, 4, 9, 11], [1, 4, 9, 13], [1, 4, 11, 13], [1, 7, 12, 13], [1, 11, 12, 13], [2, 8, 9, 13], [4, 7, 12, 13], [4, 11, 12, 13]]$$

$$[[1, 2, 10, 14], [1, 2, 12, 14], [1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 10, 14]]$$

$$\{M_0 - M_1, S M_0 - E, E M_0 + F M_0 - E, -E M_1 - F M_1 + E, -E M_3 - F M_3 + E, -E M_4 - F M_4 + E, -S M_1 - F + S, -S M_3 - F + S, -S M_4 - F + S, -E M_0 + E M_1 - F M_0 + F M_1 + S M_0 - S M_1\}$$

$$[E = 0.399860, F = 0.000202, S = 2.200280, M_0 = 0.999762, M_1 = 0.999246, M_2 = 0.999900, M_3 = 0.999246, M_4 = 0.999246]$$

$$[[0, 2, 8, 13], [0, 4, 9, 13], [0, 8, 9, 13], [1, 2, 9, 13], [1, 4, 9, 11], [1, 4, 9, 13], [1, 4, 11, 13], [1, 7, 12, 13], [1, 11, 12, 13], [2, 8, 9, 13], [4, 7, 12, 13], [4, 11, 12, 13]]$$

$$[[1, 2, 10, 14], [1, 2, 13, 14], [1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 10, 14]]$$

$$\{M_0 - M_1, S M_0 - E, E M_0 + F M_0 - E, -E M_1 - F M_1 + E, -E M_4 - F M_4 + E, -S M_1 - F + S, -S M_4 - F + S, -E M_0 + E M_1 - F M_0 + F M_1 + S M_0 - S M_1\}$$

$$[E = 0.399860, F = 0.000212, S = 2.200280, M_0 = 0.999744, M_1 = 0.999220, M_2 = 0.999900, M_3 = 0.999900, M_4 = 0.999220]$$

$$[[0, 2, 8, 13], [0, 5, 10, 13], [0, 8, 10, 13], [1, 2, 10, 13], [1, 5, 10, 12], [1, 5, 10, 13], [1, 5, 12, 13], [1, 7, 12, 13], [2, 8, 10, 13], [5, 7, 12, 13]]$$

$$[[1, 2, 11, 14], [1, 2, 13, 14], [1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 11, 14]]$$

$$\{M_0 - M_2, S M_0 - E, E M_0 + F M_0 - E, -E M_2 - F M_2 + E, -E M_4 - F M_4 + E, -S M_2 - F + S, -S M_4 - F + S, -E M_0 + E M_2 - F M_0 + F M_2 + S M_0 - S M_2\}$$

$$[E = 0.399860, F = 0.000212, S = 2.200280, M_0 = 0.999744, M_1 = 0.999900, M_2 = 0.999220, M_3 = 0.999900, M_4 = 0.999220]$$

$$[[0, 2, 8, 9], [0, 2, 9, 13], [0, 5, 10, 13], [0, 9, 10, 13], [1, 2, 10, 13], [1, 5, 10, 11], [1, 5, 10, 13], [1, 5, 11, 12], [1, 5, 12, 13], [1, 7, 12, 13], [2, 9, 10, 13], [5, 7, 12, 13]]$$

$$\begin{aligned}
& [[1, 2, 11, 14], [1, 2, 13, 14], [1, 2, 13, 14], [1, 3, 10, 14], [1, 10, 11, 14]] \\
& \{M_1 - M_2, SM_1 - E, EM_1 + FM_1 - E, -EM_2 - FM_2 + E, -EM_4 - FM_4 + E, -SM_2 - F + S, \\
& \quad -SM_4 - F + S, -EM_1 + EM_2 - FM_1 + FM_2 + SM_1 - SM_2\} \\
& [E = 0.399860, F = 0.000212, S = 2.200280, M_0 = 0.999900, M_1 = 0.999744, M_2 = 0.999220, M_3 \\
& \quad = 0.999900, M_4 = 0.999220] \\
& [[0, 2, 8, 13], [0, 4, 8, 13], [1, 2, 11, 13], [1, 6, 7, 11], [1, 6, 7, 13], [1, 6, 11, 13], [1, 7, 11, \\
& \quad 12], [2, 4, 8, 10], [2, 4, 8, 13], [2, 4, 10, 13], [2, 10, 11, 13], [4, 6, 11, 13], [4, 10, 11, 13]] \\
& \quad [[1, 2, 12, 14], [1, 3, 9, 14], [1, 3, 9, 14], [1, 3, 11, 14], [1, 11, 12, 14]] \\
& \{M_2 - M_3, SM_0 - E, SM_2 - E, EM_0 + FM_0 - E, EM_2 + FM_2 - E, -EM_3 - FM_3 + E, -SM_3 \\
& \quad - F + S, -EM_2 + EM_3 - FM_2 + FM_3 + SM_2 - SM_3\} \\
& [E = 0.399860, F = 0.000234, S = 2.200280, M_0 = 0.999900, M_1 = 0.999900, M_2 = 0.999708, M_3 \\
& \quad = 0.999166, M_4 = 0.999900] \\
& [[0, 2, 8, 13], [0, 6, 11, 13], [0, 8, 9, 13], [0, 9, 10, 13], [0, 10, 11, 13], [1, 2, 11, 13], [1, 6, 7, \\
& \quad 11], [1, 6, 7, 13], [1, 6, 11, 13], [1, 7, 11, 12], [2, 8, 9, 13], [2, 9, 10, 13], [2, 10, 11, 13]] \\
& \quad [[1, 2, 12, 14], [1, 3, 9, 14], [1, 9, 10, 14], [1, 10, 11, 14], [1, 11, 12, 14]] \\
& \{M_0 - M_1, M_2 - M_1, M_2 - M_3, SM_0 - E, EM_0 + FM_0 - E, -EM_1 - FM_1 + E, EM_2 + FM_2 \\
& \quad - E, -EM_3 - FM_3 + E, -SM_3 - F + S, -EM_0 + EM_1 - FM_0 + FM_1 + SM_0 - SM_1, \\
& \quad EM_1 - EM_2 + FM_1 - FM_2 - SM_1 + SM_2, -EM_2 + EM_3 - FM_2 + FM_3 + SM_2 - SM_3\} \\
& [E = 0.399861, F = 0.000213, S = 2.200280, M_0 = 0.999746, M_1 = 0.999217, M_2 = 0.999746, M_3 \\
& \quad = 0.999217, M_4 = 0.999900] \\
& [[0, 2, 8, 13], [0, 4, 8, 13], [1, 2, 12, 13], [1, 7, 12, 13], [2, 4, 8, 10], [2, 4, 8, 13], [2, 4, 10, \\
& \quad 13], [2, 10, 12, 13], [4, 7, 12, 13], [4, 10, 12, 13]] \\
& \quad [[1, 2, 13, 14], [1, 3, 9, 14], [1, 3, 9, 14], [1, 3, 11, 14], [1, 11, 13, 14]] \\
& \{M_2 - M_4, SM_0 - E, SM_2 - E, EM_0 + FM_0 - E, EM_2 + FM_2 - E, -EM_4 - FM_4 + E, -SM_4 \\
& \quad - F + S, -EM_2 + EM_4 - FM_2 + FM_4 + SM_2 - SM_4\} \\
& [E = 0.399860, F = 0.000234, S = 2.200280, M_0 = 0.999900, M_1 = 0.999900, M_2 = 0.999708, M_3 \\
& \quad = 0.999900, M_4 = 0.999166] \\
& [[0, 2, 8, 13], [0, 5, 9, 13], [0, 8, 9, 13], [1, 2, 12, 13], [1, 7, 12, 13], [2, 5, 9, 11], [2, 5, 9, \\
& \quad 13], [2, 5, 11, 13], [2, 8, 9, 13], [2, 11, 12, 13], [5, 7, 12, 13], [5, 11, 12, 13]] \\
& \quad [[1, 2, 13, 14], [1, 3, 9, 14], [1, 3, 10, 14], [1, 3, 12, 14], [1, 12, 13, 14]] \\
& \{M_3 - M_4, SM_0 - E, SM_1 - E, SM_3 - E, EM_0 + FM_0 - E, EM_1 + FM_1 - E, EM_3 + FM_3 \\
& \quad - E, -EM_4 - FM_4 + E, -SM_4 - F + S, -EM_3 + EM_4 - FM_3 + FM_4 + SM_3 - SM_4\} \\
& [E = 0.399860, F = 0.000234, S = 2.200280, M_0 = 0.999900, M_1 = 0.999900, M_2 = 0.999900, M_3 \\
& \quad = 0.999708, M_4 = 0.999166] \\
& [[0, 2, 8, 13], [0, 6, 10, 13], [0, 8, 9, 13], [0, 9, 10, 13], [1, 2, 12, 13], [1, 7, 12, 13], [2, 6, 10,
\end{aligned}$$

12], [2, 6, 10, 13], [2, 6, 12, 13], [2, 8, 9, 13], [2, 9, 10, 13], [6, 7, 12, 13]]

[[1, 2, 13, 14], [1, 3, 9, 14], [1, 3, 11, 14], [1, 9, 10, 14], [1, 10, 11, 14]]

$$\{M_0 - M_1, M_2 - M_1, S M_0 - E, S M_2 - E, E M_0 + F M_0 - E, -E M_1 - F M_1 + E, E M_2 + F M_2 - E, -E M_4 - F M_4 + E, -S M_4 - F + S, -E M_0 + E M_1 - F M_0 + F M_1 + S M_0 - S M_1, E M_1 - E M_2 + F M_1 - F M_2 - S M_1 + S M_2\}$$

$$[E = 0.437321, F = 0.053049, S = 2.275562, M_0 = 0.936458, M_1 = 0.891541, M_2 = 0.940877, M_3 = 0.999900, M_4 = 0.874851]$$

[[0, 2, 8, 13], [0, 7, 12, 13], [0, 8, 9, 13], [0, 9, 10, 13], [0, 10, 12, 13], [1, 2, 12, 13], [1, 7, 12, 13], [2, 8, 9, 13], [2, 9, 10, 13], [2, 10, 12, 13]]

[[1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 10, 14], [1, 10, 11, 14], [1, 11, 13, 14]]

$$\{M_0 - M_1, M_2 - M_1, M_2 - M_4, S M_0 - E, E M_0 + F M_0 - E, -E M_1 - F M_1 + E, E M_2 + F M_2 - E, -E M_4 - F M_4 + E, -S M_4 - F + S, -E M_0 + E M_1 - F M_0 + F M_1 + S M_0 - S M_1, E M_1 - E M_2 + F M_1 - F M_2 - S M_1 + S M_2, -E M_2 + E M_4 - F M_2 + F M_4 + S M_2 - S M_4\}$$

$$[E = 0.399861, F = 0.000213, S = 2.200280, M_0 = 0.999746, M_1 = 0.999217, M_2 = 0.999746, M_3 = 0.999900, M_4 = 0.999217]$$

[[0, 2, 8, 13], [0, 7, 12, 13], [0, 8, 9, 13], [0, 9, 11, 13], [0, 11, 12, 13], [1, 2, 12, 13], [1, 7, 12, 13], [2, 8, 9, 13], [2, 9, 11, 13], [2, 11, 12, 13]]

[[1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 10, 14], [1, 10, 12, 14], [1, 12, 13, 14]]

$$\{M_0 - M_1, M_3 - M_1, M_3 - M_4, S M_0 - E, E M_0 + F M_0 - E, -E M_1 - F M_1 + E, E M_3 + F M_3 - E, -E M_4 - F M_4 + E, -S M_4 - F + S, -E M_0 + E M_1 - F M_0 + F M_1 + S M_0 - S M_1, E M_1 - E M_3 + F M_1 - F M_3 - S M_1 + S M_3, -E M_3 + E M_4 - F M_3 + F M_4 + S M_3 - S M_4\}$$

$$[E = 0.399861, F = 0.000213, S = 2.200280, M_0 = 0.999746, M_1 = 0.999217, M_2 = 0.999900, M_3 = 0.999746, M_4 = 0.999217]$$

[[0, 2, 8, 13], [0, 7, 12, 13], [0, 8, 10, 13], [0, 10, 11, 13], [0, 11, 12, 13], [1, 2, 12, 13], [1, 7, 12, 13], [2, 8, 10, 13], [2, 10, 11, 13], [2, 11, 12, 13]]

[[1, 2, 13, 14], [1, 3, 9, 14], [1, 9, 11, 14], [1, 11, 12, 14], [1, 12, 13, 14]]

$$\{M_0 - M_2, M_3 - M_2, M_3 - M_4, S M_0 - E, E M_0 + F M_0 - E, -E M_2 - F M_2 + E, E M_3 + F M_3 - E, -E M_4 - F M_4 + E, -S M_4 - F + S, -E M_0 + E M_2 - F M_0 + F M_2 + S M_0 - S M_2, E M_2 - E M_3 + F M_2 - F M_3 - S M_2 + S M_3, -E M_3 + E M_4 - F M_3 + F M_4 + S M_3 - S M_4\}$$

$$[E = 0.399861, F = 0.000213, S = 2.200280, M_0 = 0.999746, M_1 = 0.999900, M_2 = 0.999217, M_3 = 0.999746, M_4 = 0.999217]$$

[[0, 2, 8, 13], [0, 5, 9, 13], [0, 8, 9, 13], [1, 2, 12, 13], [1, 7, 12, 13], [2, 5, 9, 11], [2, 5, 9, 13], [2, 5, 11, 13], [2, 8, 9, 13], [2, 11, 12, 13], [5, 7, 12, 13], [5, 11, 12, 13]]

[[1, 2, 13, 14], [1, 3, 10, 14], [1, 3, 10, 14], [1, 3, 12, 14], [1, 12, 13, 14]]

$$\{M_3 - M_4, S M_1 - E, S M_3 - E, E M_1 + F M_1 - E, E M_3 + F M_3 - E, -E M_4 - F M_4 + E, -S M_4$$

$$\begin{aligned}
& -F + S, -EM_3 + EM_4 - FM_3 + FM_4 + SM_3 - SM_4\} \\
& [E = 0.399860, F = 0.000234, S = 2.200280, M_0 = 0.999900, M_1 = 0.999900, M_2 = 0.999900, M_3 \\
& = 0.999708, M_4 = 0.999166] \\
& [[0, 2, 8, 9], [0, 2, 9, 13], [0, 7, 12, 13], [0, 9, 10, 13], [0, 10, 11, 13], [0, 11, 12, 13], [1, 2, \\
& 12, 13], [1, 7, 12, 13], [2, 9, 10, 13], [2, 10, 11, 13], [2, 11, 12, 13]] \\
& [[1, 2, 13, 14], [1, 3, 10, 14], [1, 10, 11, 14], [1, 11, 12, 14], [1, 12, 13, 14]] \\
& \{M_1 - M_2, M_3 - M_2, M_3 - M_4, SM_1 - E, EM_1 + FM_1 - E, -EM_2 - FM_2 + E, EM_3 + FM_3 \\
& - E, -EM_4 - FM_4 + E, -SM_4 - F + S, -EM_1 + EM_2 - FM_1 + FM_2 + SM_1 - SM_2, \\
& EM_2 - EM_3 + FM_2 - FM_3 - SM_2 + SM_3, -EM_3 + EM_4 - FM_3 + FM_4 + SM_3 - SM_4\} \\
& [E = 0.399861, F = 0.000213, S = 2.200280, M_0 = 0.999900, M_1 = 0.999746, M_2 = 0.999217, M_3 \\
& = 0.999746, M_4 = 0.999217]
\end{aligned}$$

(5)