Arreglos de hiperplanos

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Encuentro Rioplatense de Álgebra y Geometría Algebraica
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Deconificación

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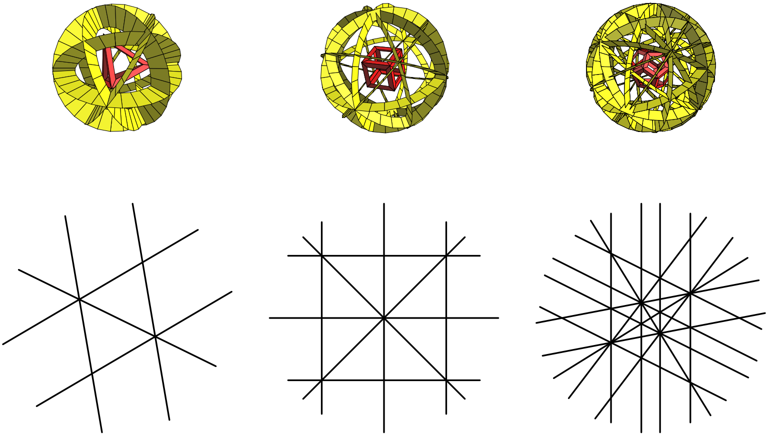
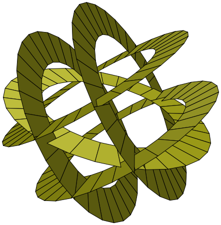
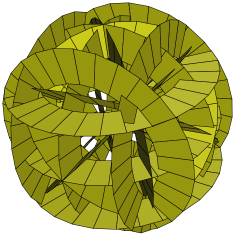
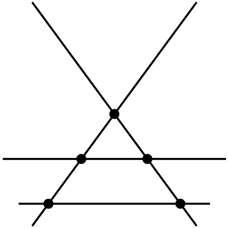


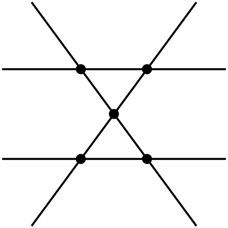
Deconificación

Equivalencia combinatoria



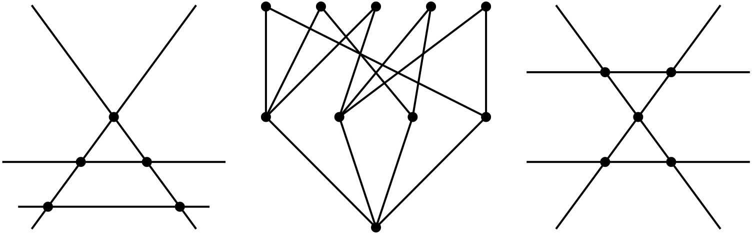
$\mathcal{A}$



$\mathcal{A}'$

Dos arreglos no afínmente equivalentes con el mismo poset de intersección.

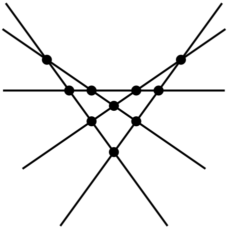
Equivalencia combinatoria

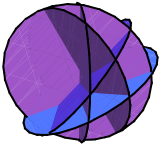


$\mathcal{A}$ $\quad L(\mathcal{A}) \cong L(\mathcal{A}')$ $\quad \mathcal{A}'$

Dos arreglos no afínmente equivalentes con el mismo poset de intersección.

Arreglos en posición general

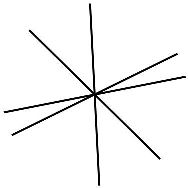




Racionalidad

Racionalidad

$$Q(\mathcal{A}) = \prod_{i=1}^{4}(a_i x - b_i y)$$



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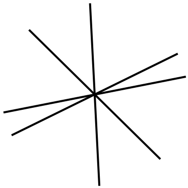

$$P_1 = (a_1 : b_1), \ldots, P_4 = (a_4 : b_4) \in \mathbb{P}^1(\mathbb{k})$$

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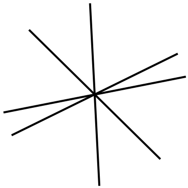
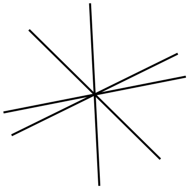


$$P_1 = (a_1 : b_1), \ldots, P_4 = (a_4 : b_4) \in \mathbb{P}^1(\Bbbk)$$

$$\lambda = (P_1, P_2; P_3, P_4) = \frac{\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \begin{vmatrix} a_2 & a_4 \\ b_2 & b_4 \end{vmatrix}}{\begin{vmatrix} a_1 & a_4 \\ b_1 & b_4 \end{vmatrix} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}}$$

Racionalidad

$Q(\mathcal{A}) = \prod_{i=1}^{4}(a_i x - b_i y)$



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$$\rho(\mathcal{A}) = \{\lambda, \frac{1}{\lambda}, \frac{1}{1-\lambda}, 1-\lambda, \frac{\lambda}{1-\lambda}, \frac{1-\lambda}{\lambda}\} \subseteq \mathbb{k} \cup \{\infty\}.$$

Racionalidad

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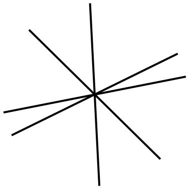

$$Q(\mathcal{A}) = xy(x+y)(x\sqrt{2}-y)$$

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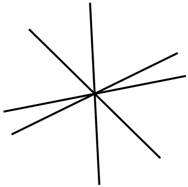

$$Q(\mathcal{A}) = xy(x+y)(x\sqrt{2}-y)$$

$$\rho(\mathcal{A}) \ni \sqrt{2}$$

Racionalidad

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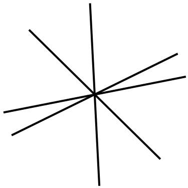

$$Q(\mathcal{A}) = xy(x+y)(x\sqrt{2} - y)$$

$$\rho(\mathcal{A}) \ni \sqrt{2}$$

$\therefore \mathcal{A}$ no es racional sobre $\mathbb{Q}$

Racionalidad: tipos combinatorios

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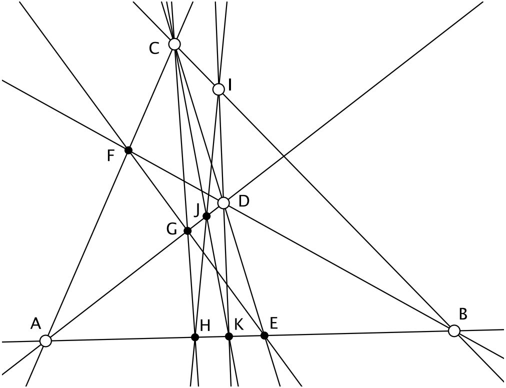
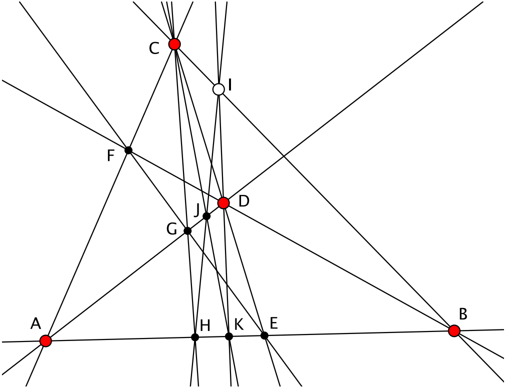


Un tipo combinatorio definido sobre $\mathbb{Q}(\sqrt{2})$ pero no sobre $\mathbb{Q}$

Racionalidad: tipos combinatorios



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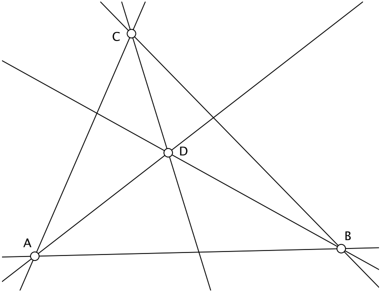
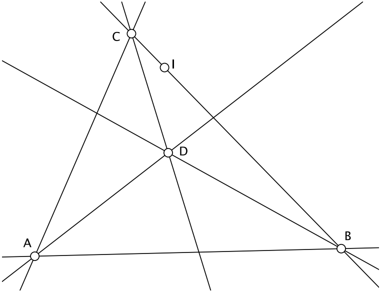


$$A = [1 : 0 : 0]$$
$$B = [0 : 1 : 0]$$
$$C = [0 : 0 : 1]$$
$$D = [1 : 1 : 1]$$

Racionalidad: tipos combinatorios



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$$D = [1 : 1 : 1]$$
$$I = [0 : 1 : \lambda]$$

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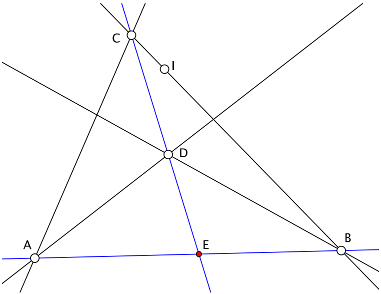


$A = [1 : 0 : 0]$

$B = [0 : 1 : 0]$

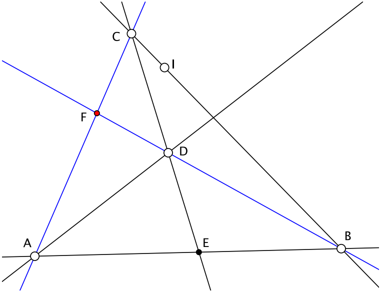
$C = [0 : 0 : 1]$

$D = [1 : 1 : 1]$

$I = [0 : 1 : \lambda]$

$E = AB \cap CD = [1 : 1 : 0]$

Racionalidad: tipos combinatorios



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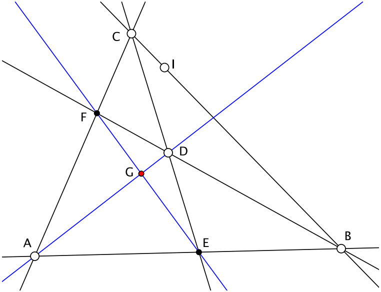
Racionalidad: tipos combinatorios



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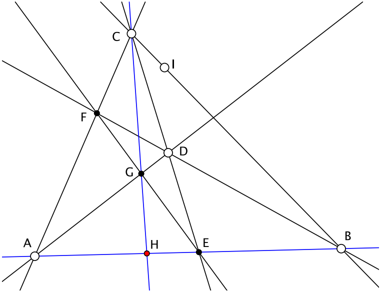
Racionalidad: tipos combinatorios



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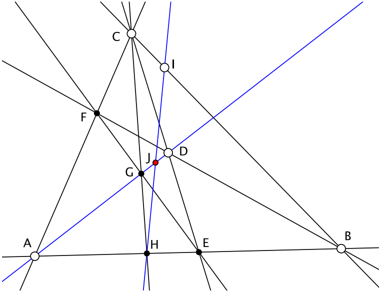
$$C = [0:0:1]$$
$$D = [1:1:1]$$
$$I = [0:1:\lambda]$$
$$E = AB \cap CD = [1:1:0]$$
$$F = AC \cap BD = [1:0:1]$$
$$G = AD \cap EF = [2:1:1]$$
$$H = AB \cap CG = [2:1:0]$$

Racionalidad: tipos combinatorios



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$$J = AD \cap HI = [2\lambda - 2 : \lambda : \lambda]$$

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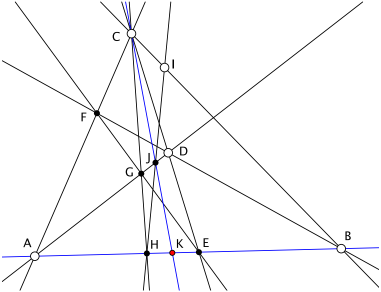

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$$J = AD \cap HI = [2\lambda - 2 : \lambda : \lambda]$$
$$K = AB \cap CJ = [2\lambda - 2 : \lambda : 0]$$

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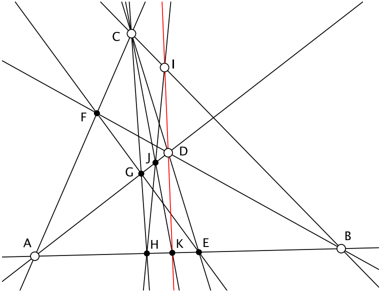

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$$J = AD \cap HI = [2\lambda - 2 : \lambda : \lambda]$$
$$K = AB \cap CJ = [2\lambda - 2 : \lambda : 0]$$
$$DI = [\lambda - 1 : -\lambda : 1]$$

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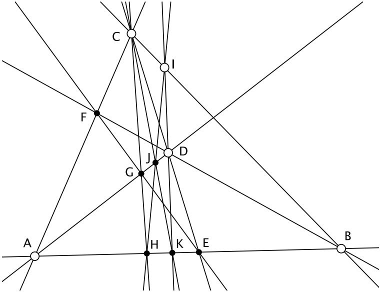

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$$DI = [\lambda - 1 : -\lambda : 1]$$

$$K \in DI \iff \lambda^2 - 4\lambda - 2 = 0 \iff (\lambda - 2)^2 = 2$$

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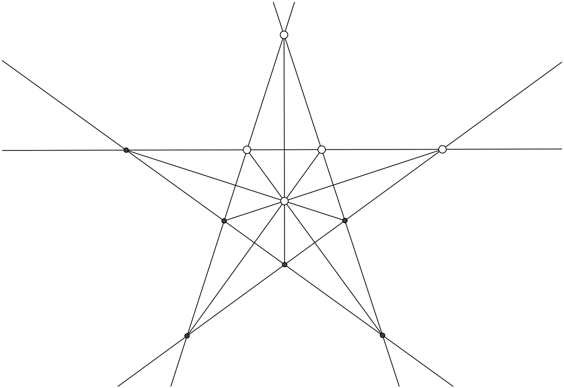

Un tipo combinatorio definido sobre $\mathbb{Q}(\sqrt{5})$ pero no sobre $\mathbb{Q}$