AN INTRODUCTION TO THE KPZ EQUATION

The aim of this course is to give an introduction to what is called the KPZ equation. The KPZ equation was introduced in [3] as a continuous model for stochastic growth of interfaces and corresponds to the stochastic PDE

$$\partial_t h = \nu \partial_{xx} h + a\big(\partial_x h\big)^2 + \sigma \mathcal{W},$$

where $h : \mathbb{R} \times [0, \infty) \to \mathbb{R}$ is a space-time stochastic process and $\mathcal{W}$ is a standard white noise in $\mathbb{R} \times [0, \infty)$. Up to date, existence and uniqueness of solutions, as well as well-poseness of the Cauchy problem are still missing. In the original work of [3], a notion of solution, known as the *Cole-Hopf solution* was proposed. The idea is use the Cole-Hopf transformation at a formal level to linearize the KPZ equation. This notion of solution leads to existence, uniqueness and well-posedness, although the validity of the formal transformation is still to be justified in a rigorous way. It was not until ten years ago that it was proven that this notion of solution arises in a natural way from a particular lattice dynamics: the so-called asymmetric simple exclusion process. Recently, deep connections between the KPZ equation, random polymers and also random matrices were discovered, which gathered a revival of the research around the KPZ equation. However, existence, uniqueness and well-posedness still remain as one of the main open problems of the field.

This course will consists of three lectures; a temptative program is the following:

Lecture 1: The KPZ equation and the stochastic Burgers equation; the Cole-Hopf transformation and the stochastic heat equation. Existence and uniqueness of solutions of the stochastic heat equation.

Lecture 2: The asymmetric simple exclusion process; Gartner transformation and convergence to the stochastic heat equation. Bertini-Giacomin theorem [1].

Lecture 3: Definition of energy solutions of the KPZ equation. Universality of stationary energy solutions of the KPZ equation.

References

[1] L. Bertini and G. Giacomin, *Stochastic Burgers and KPZ equations from particle systems*, Comm. Math. Phys. **183**(3), 571–607 (1997).

[2] Patrícia Gonçalves and Milton Jara, *Universality of KPZ equation*, (2010), `http://arxiv.org/abs/1003.4478`

[3] M. Kardar, G. Parisi and Y.-C. Zhang, *Dynamic Scaling of Growing Interfaces*, Phys. Rev. Lett. **56**(9), 889–892 (Mar 1986).