

## On Sentis Solution to Discontinuous Differential Equation

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### Abstract

Let  $f : R^n \rightarrow R^n$  be given. The Fillipov of  $f$  is defined as follows

$$F[f](x) = \bigcap_{\varepsilon > 0} \bigcap_{m(N)=0} \overline{\text{conv}}f(x + \varepsilon B \setminus N), \quad (*)$$

where  $m$  denotes the Lebesgue measure,  $\overline{\text{conv}}A$  represents the closure of the convex hull of the set  $A$  and  $x + \varepsilon B$  represents the open ball of radius  $\varepsilon$  about the point  $x$ . The Fillipov is used in defining a generalized solution to the ordinary differential equation  $x' = f(t, x)$ , particularly in the case of  $f$  is discontinuous in  $x$ .

It is commonly recongized that Fillipov solutions are “too many” for some applications, so to overcome this drawback, Sentis replaces (\*) to the differential inclusion

$$\dot{x}(t) \in F(x(t)) \text{ a.e., } t \in [0, T] \text{ by } F_s(x) = \bigcap_{\varepsilon > 0} \bigcap_{m(N)=0} \overline{\{f(x + \varepsilon B \setminus N)\}}$$

The set valued map  $F_s(x)$  turn out to be upper semicontinuous, locally bounded and compact, but in general not convex.

In this paper, we study the properties of the set valued map  $F_s(x)$ . Also, we compare the Sentis solution with the other famous generalized solutions to the discontinuous differential equation.

## Orbital stability of periodic traveling waves for Benjamin-Bona-Mahony equation with fractional nonlinear terms

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### Abstract

In this poster we deal with orbital stability of periodic traveling waves for dispersive models. The study of traveling waves started in the mid-18th century when John S. Russel established that the flow of water waves in a shallow channel has constant evolution. The general strategy to obtain stability consists in proving that the traveling wave in question minimizes a conserved functional restricted to a certain manifold. In our context, following such ideas, we minimize such a functional restricted to a new manifold. Although our theory can be applied to other models, here we apply it to periodic traveling wave solutions of Benjamin- Bona-Mahony equation with fractional nonlinear terms

$$u_t - u_{xxt} + (u + |u|^{\frac{3}{2}})_x = 0.$$

## Boundary control for a system of wave equations

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### Abstract

We study exact boundary control for the constant coefficient system of coupled wave equations

$$\begin{aligned}u_\nu \Delta u + \alpha u + \beta v &= 0, \\v_\nu \Delta v + \gamma u + \delta v &= 0,\end{aligned}$$

in piecewise smooth bounded domains. We show that under the action of square integrable boundary controls of Neuman type, initial states with finite energy may be driven to rest after finite time  $T$ . The controls may act on the whole boundary or on suitable parts of it. We use decay of energy to obtain controllability and spectral theory to minimize  $T$ .

## Convergence analysis of a re-scaled nonlocal diffusion equation

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### Abstract

We study the convergence of a re-scaled nonlocal diffusion. We show that the solution converges to the solution of the porous medium equation when the parameter goes to zero.

## Functions of bounded $\Phi$ -variation on compact subsets of $\mathbb{C}$

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### Abstract

In this paper we introduce the concept of bounded  $\Phi$ -variation for functions defined on compact subsets of the complex plane  $\mathbb{C}$ , based on the notion of variation on paths as defined by Ashton and Doust (*Functions of bounded variation on compact subsets of the plane*, *Studia Math.*, 169 (2005), 163-188). We present a generalization of the concept of function of bounded variation for functions defined on compact subsets of the complex plane  $\mathbb{C}$  presented for J. Giménez, N. Merentes and M. Vivas (*Functions of bounded variation on compact subsets of  $\mathbb{C}$* , *Commentationes Mathematicae*, Vol 54 N 1, 3-19 (2014)). We describe in detail the space so generated and show that it can be equipped with the structure of a Banach space. We also present a necessary condition for a composition operator  $C_\varphi$  to act on a such space.

## Approximate Controllability of Semilinear System of Parabolic Equations with Delay

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### Abstract

In this paper we prove under certain conditions the approximate controllability of a semilinear system parabolic equations with delay.

## Convergence and Derivation of Stochastic Reaction-Diffusion Equations

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### Abstract

In the present work, we study the stochastic reaction-diffusion equation on  $[0, 1]$ , with Dirichlet boundary and perturbed by the space-time white noise, that is:

$$\begin{aligned} dX(t, \xi) &= \left[ \frac{\partial^2}{\partial \xi^2} X(t, \xi) + f(X(t, \xi)) \right] dt + dW(t, \xi), \\ X(t, 0) &= X(t, 1) = 0, \quad t > 0 \\ X(0, \xi) &= x_0(\xi), \quad \xi \in ]0, 1[. \end{aligned}$$

where  $W$  is the Brownian sheet. We approximate the random perturbation through of a process of regularization (convolution) depending on a parameter  $k \in \mathbb{N}$ . Then, we will provide conditions necessities so that the mild solutions associated to the regularized stochastic partial differential equation (SPDE) converges to the mild.

## Regularity up to the boundary for singularly perturbed fully nonlinear elliptic equations

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### Abstract

In this exhibition we are interested in studying regularity up to the boundary for one-phase singularly perturbed fully nonlinear elliptic problems, associated to high energy activation potentials, namely

$$F(X, \nabla u^\varepsilon, D^2 u^\varepsilon) = \zeta_\varepsilon(u^\varepsilon), \quad \text{in } \Omega \subset \mathbb{R}^n$$

where  $\zeta_\varepsilon$  behaves asymptotically as the Dirac measure  $\delta_0$  as  $\varepsilon$  goes to zero. We shall establish global gradient bounds independent of the parameter  $\varepsilon$ . Keywords: Fully nonlinear elliptic operators, one-phase problems, regularity up to the boundary, singularly perturbed equations. This work was done together with Gleydson Chaves Ricarte.

## Generalized Riemann problem for the Suliciu relaxation system

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### Abstract

we show explicitly the solution of the generalized Riemann problem to better understand the explicit solutions. First, we consider the generalized Riemann problem for the Suliciu relaxation system in Lagrangian coordinates and we calculate the first-order expansion given by LeFloch and Raviart to verify our results. After, we show the explicit solution for the generalized Riemann problem in Eulerian coordinates and since the solution has a similar structure as the classical Riemann problem, we give an example for the interaction of elementary waves.

## On Bi-dimensional Second $\mu$ -Variation

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### Abstract

In this paper we present a generalization of the notion of bounded slope variation for functions defined on a rectangle  $I_a^b$  in  $\mathbb{R}^2$ . Given a strictly increasing function  $\mu$ , defined in a closed real interval, we introduce the class  $BV^{\mu,2}(I_a^b)$ , of functions of *bounded second  $\mu$ -variation* on  $I_a^b$ , and show that this class can be equipped with a norm with respect to which it is a Banach space. We also deal with the important case of factorizable functions in  $BV^{\mu,2}(I_a^b)$  and finally we exhibit a relation between this class and the one of double Riemann-Stieltjes integrals of functions of bi-dimensional bounded variation.

## Continuous data assimilation for Navier-Stokes-alpha model

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### Abstract

Motivated by the presence of a finite number of determining parameters (degrees of freedom) such as modes, nodes and local spatial averages for dissipative dynamical systems, specially Navier-Stokes equations, we present in this work a new continuous data assimilation algorithm for the three-dimensional Navier-Stokes- $\alpha$  model, which consists of introducing a general type of approximation interpolation operator, (that is constructed from observational measurements), into this model. equations. The main result provides conditions on the finite- dimensional spatial resolution of the collected data, sufficient to guarantee that the approximating solution, that is obtained from these collected data, converges to the unknown solution over time.

## The time-fractional diffusion-wave equation in intermediate diffusion phenomena

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### Abstract

An initial-boundary-value problem for the time-fractional diffusion-wave equation in the upper half-plane is studied, where the fractional derivative is taken in Caputo sense of order  $\alpha \in (0, 2)$ . These kind of problems are strongly related to the behaviour of linear viscoelastic media subject to stress pulses, since such processes are intermediate between diffusion and wave propagation. The solution to this problem is obtained; as well as the first proof of this result. Different properties of the solution to this problem, such as its asymptotic behaviour and the limiting behaviour  $\alpha \rightarrow 1$  and  $\alpha \rightarrow 2^-$ , are presented, together with their connection to the physical interpretation of this problem.

## The Zakharov-Kuznetsov equation in weighted Sobolev spaces

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### Abstract

In this talk we consider the initial value problem (IVP) associated to the two dimensional Zakharov-Kuznetsov (ZK) equation,

$$\left. \begin{aligned} u_t + \partial_x^3 u + \partial_x^2 \partial_y u + u \partial_x u &= 0, & (x, y) \in \mathbb{R}^2, t \in \mathbb{R}, \\ u(x, y, 0) &= u_0(x, y). \end{aligned} \right\} \quad (1)$$

In this work we will be concerned with the well-posedness of the IVP (1) in weighted Sobolev spaces. We will study real valued solutions of the IVP (1) in the weighted Sobolev spaces

$$Z_{s,r} := H^s(\mathbb{R}^2) \cap L^2((1+x^2+y^2)^r dx dy),$$

with  $s, r \in \mathbb{R}$ .

Our aim in this lecture is to prove that the IVP (1) is locally well-posedness in  $Z_{s,s/2}$  for  $s > 3/4$ .

## The Cauchy problem associated to a neural field model with hebbian-type synaptic modification

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### Abstract

Neural fields are gaining momentum in the theoretical neuroscience community as description of mesoscopic activity of the brain. They also are source of interesting mathematical problem being them, in general, nonlinear, sometimes singular, integrodifferential equations. In this work we consider the

model of Abbassian et. al, for a spatially structured neural network or Neural Field, with Hebbian like synaptic modification

$$\begin{cases} u_t(x, t) = -u + \int_{\Omega} w(x, y)G(u(x, t), u(y, t), \gamma)f(u(y, t)) dy, & (x, t) \in \Omega \times (0, \infty) \\ u(x, 0) = u_0(x), & x \in \Omega \subseteq \mathbb{R}^m, \end{cases} \quad (1)$$

with

$$G(u(x, t), u(y, t), \gamma) = 1 + \gamma g(u(x, t) - u(y, t)), \quad (2)$$

$\gamma > 0$ , and we show well-posedness in  $C_b(\Omega)$  for  $\Omega = \mathbb{R}^m$  and in  $L^1(\Omega)$  for  $\Omega \subset \mathbb{R}^m$ , bounded, with the synaptic kernel  $w$  and the functions  $f, g$ , bounded, with bounded derivative. We show that in the limit  $\gamma \rightarrow 0$ , the solutions to the model (1) converge uniformly to the solution of the Amari's classical model

$$\begin{cases} u_t(x, t) = -u(x, t) + \int_{\Omega} w(x, y)f(u(y, t)) dy, & (x, t) \in \Omega \times (0, \infty) \\ u(x, 0) = u_0(x), & x \in \Omega \subseteq \mathbb{R}^m, \end{cases} \quad (3)$$

studied by other authors

## Decay Of Solutions To Cauchy Viscoelastic Problems With Density

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### Abstract

The following semilinear viscoelastic equation

$$u_{tt} - \Delta u(x) + \int_0^t g(t-s)\Delta u(x, s)ds + |u|^\gamma u = 0,$$

has been studied in a bounded domain, and a uniform decay result was established.

Here, we would like to extend this result of decay from bounded domains to problems in  $\mathbb{R}^n$ . In this case, Poincaré's inequality, and some Lebesgue and Sobolev embedding inequalities are no longer valid. To overcome this difficulty, we need to use some weighted spaces.

More precisely, we look into problems of the type

$$\rho(x)u_{tt} - \Delta u(x) + \int_0^t g(t-s)\Delta u(x, s)ds = 0, \quad x \in \mathbb{R}^n, \quad t > 0,$$

$$u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x), \quad x \in \mathbb{R}^n,$$

and show general decays rates. This study includes the exponential and polynomial rates as particular cases.

## Approximate Controllability of Semilinear Impulsive Strongly Damped Wave Equation

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### Abstract

Rothe's fixed point theorem is applied to prove the interior approximate controllability of the following semilinear impulsive strongly damped wave equation with Dirichlet boundary conditions. Under some conditions we prove the following statement: For all open nonempty subsets  $\omega$  of  $\Omega$  the system is approximately controllable on  $[0, \tau]$ . Moreover, we exhibit a sequence of controls steering the nonlinear system from an initial state  $z_0$  to an  $\epsilon$ -neighborhood of the final state  $z_1$  at time  $\tau > 0$ .

## Factores integrantes en la teoria de funciones de variacion acotada generalizada

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### Abstract

discutiremos algunos aspectos relacionados con, el así llamado, *Problema del Operador de Superposición* (P.O.S) en espacios de funciones con algún tipo de variación acotada. Mostraremos el rol fundamental que juegan las propiedades intrínsecas de la función interior involucrada en la composición de dos funciones pertenecientes a las clases mencionadas. En particular, mostraremos que si  $g$  es una función integrable entonces su multiplicación por una función  $f$  con derivada continua mejora las propiedades de integrabilidad de la composición  $g \circ f$ .

## Dynamics of parabolic problems under discretization of finite element method

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### Abstract

Consider the initial boundary value problem of parabolic type

$$\begin{cases} u_t = Lu + f(u), & t > 0, x \in \Omega \\ u = 0, & t > 0, x \in \partial\Omega, \\ u(x, 0) = \varphi(x) \end{cases} \quad (1)$$

where  $\varphi \in H_0^1(\Omega)$ ,  $L$  is a second order uniformly strongly elliptic operator and  $f$  is dissipative nonlinearity. In this work we will treat about the continuity of the of equilibria as the step size goes to zero for the discretization of parabolic problem (1) using the finite element method.

## Some existence results on periodic solutions of Euler-Lagrange equations in an Orlicz-Sobolev space setting

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### Abstract

In this work we consider the problem of finding periodic solutions of certain Euler-Lagrange equations. We employ the direct method of the calculus of variations, this is we obtain solutions minimizing certain functional  $I$ . We give conditions which ensure that  $I$  is defined and it is differentiable on certain subsets of Orlicz-Sobolev spaces  $W^1L$  associated to a  $N$ -function  $\Phi$ . We discuss various conditions for the coercivity of  $I$ . We show that, in some sense, it is necessary for the coercivity that the complementary function of  $\Phi$  be a  $\Delta_2$  function. We conclude by discussing conditions for existence of minima for  $I$ .

## Relative Asymptotic Equivalence of Difference Equations

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### Abstract

In this paper we study the relative asymptotic equivalence between the solutions of the following two difference equations in a Banach space  $Z$

$$y(n+1) = A(n)y(n), \quad x(n+1) = A(n)x(n) + f(n, x(n)),$$

where  $y(n), x(n) \in Z$ ,  $A \in l^\infty(\mathbb{N}, L(Z))$  and the function  $f : \mathbb{N} \times Z \rightarrow Z$  is small enough in some sense. The generalized discrete dichotomy definition and a discrete version of Rodrigues Inequality play the main toll reaching our results, which is the following: Given a solution  $y(n)$  of the unperturbed system, we provide sufficient conditions to prove that there exist a family of solutions  $x(n)$  for the perturbed system such that

$$\|y(n) - x(n)\| = o(\|y(n)\|), \quad \text{as } n \rightarrow \infty.$$

Conversely, given a solution  $x(n)$  of the perturbed system having Lyapunov number  $\alpha \in \mathbb{R}$ , we prove that, under certain conditions, there exist a family of solutions  $y(n)$  for the unperturbed system, such that

$$\|y(n) - x(n)\| = o(\|x(n)\|), \quad \text{as } n \rightarrow \infty.$$



## Controllability for difference equations of semilinear stochastic systems

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### Abstract

In this paper we study the exact controllability of the following difference equation of semilinear stochastic system

$$z(n+1, \omega) = A(n)z(n, \omega) + B(n)u(n, \omega) + f(z(n, \omega), u(n, \omega)) \\ + \{\Sigma(n, \omega) + \sigma(z(n, \omega), u(n, \omega))\}(w(n, \omega) - w(n-1, \omega)), \quad n \in \mathbb{N},$$

being  $Z$ ,  $U$  and  $\mathcal{E}$  Hilbert spaces,  $z(n) \in Z$ ,  $\mathbb{N}^* = \mathbb{N} \cup \{0\}$ ,  $A \in l^\infty(\mathbb{N}, L(Z))$ ,  $B \in l^\infty(\mathbb{N}, L(U, Z))$ , the control  $u$  is a stochastic process in the Hilbert space  $U$ , and the set  $\{w(n, \omega)\}_{n \in \mathbb{N}}$  is a Wiener process on  $\mathcal{E}$ .

## The scalar Chern-Simons equation

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### Abstract

We investigate the scalar Chern-Simons equation

$$-\Delta u + e^u(e^u - 1) - 1 = \mu \quad \Omega$$

with Dirichlet condition, where  $\mu$  is a Radon measure in  $\Omega$ , in terms of stability of solutions. The above problem has always solution for measures whose values on unitary sets do not exceed  $2\pi$ . Here we are interested in understanding what happens when we force the problem to have solution by approximating the datum  $\mu$  in cases where the equation has not solution. In the presentation, we will describe the limit of the sequence of solutions of the approximated problems in terms of  $\mu$ .

## On the controllability of descriptor impulsive systems

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### Abstract

In this paper, the controllability of descriptor semilinear impulsive nonautonomous systems is studied. The controllability condition is proved by applying the Rothe's type fixed point theorem for semilinear impulsive nonautonomous systems, which is obtained by transformation of the original descriptor system, from linear injective application. Then, the controllability of the semilinear impulsive nonautonomous systems is equivalent to the controllability condition for the descriptor semilinear impulsive system.

## A Generalized Neumann Solution for the Two-Phase Fractional Lam'-Clapeyron-Stefan Problem

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### Abstract

A generalized Neumann solution for the two-phase fractional Lam'-Clapeyron-Stefan problem for a semi-infinite material with constant boundary and initial conditions is obtained. In this problem, the two governing equations and a governing condition for the free boundary include a fractional time derivative in the Caputo sense of order  $0 < \alpha \leq 1$ . When  $\alpha \nearrow 1$  the classical Neumann solution for the two-phase Lam'-Clapeyron-Stefan problem given through the error function is recovered.

## On a control problem associated to the Rayleigh-Bnard-Marangoni system

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### Abstract

A boundary control problem associated to the stationary Rayleigh-Bnard-Marangoni (RBM) system is studied. Controls for the velocity and the temperature on parts of the boundary are considered, and the existence of optimal solutions is proved. By using the principle of Lagrange multipliers an optimality system is derived. By the way, the existence, uniqueness and regularity of weak solutions for the stationary RBM system in a polyhedral domain of  $R^3$  is analyzed. In particular, we deal with an elliptic regularity problem with mixed (Dirichlet, Neumann and Robin) boundary data on polyhedral domains. These results were obtained in [1].

[1] D. A. Rueda-Gomez and E. J. Villamizar-Roa. An optimal control problem for Rayleigh-Bnard-Marangoni model. (Submitted 2014).

## Parabolic equation with term nonlinearity and optimal regularity

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### Abstract

Our goal is to investigate a model of parabolic equation with singular nonlinearity related with unstable free boundary problems. Given a bounded smooth domain  $\Omega \subset \mathbb{R}^N$  and the associated local space-time domain  $Q_T = \Omega \times (0, T]$ ,  $T > 0$ , we fix two prescribed functions  $u_0 \in L^\infty(\Omega)$  and  $g \in W_2^{1,1}(Q_T) \cap L^\infty(Q_T)$ ,  $u_0, g \geq 0$ . We consider the problem

$$\begin{cases} u_t - \Delta u &= \chi_{\{u>0\}} f(u) & \text{in } Q_T, \\ u &= g & \text{on } \partial\Omega \times (0, T], \\ u &= u_0 & \text{in } \Omega \times \{0\}. \end{cases} \quad (4)$$

Existence of solution to the above problem is obtained by a disturbance of the same and for this we obtain uniform in  $\epsilon > 0$  estimates are proved in order to pass to the limit as  $\epsilon \rightarrow 0^+$  to obtain a candidate for a solution of (4). We obtain estimates uniform in  $\epsilon > 0$   $L^\infty$  that is essential to obtain compactness. Also we obtain a weighted local estimate of the gradient and in time of the approximate solution  $u^\epsilon$  in  $\Omega$ . Finally, as focus of this work prove the existence of solution for this problem and study the regularity in order to obtain optimal regularity.

## Convergence results of the best Sobolev trace constant and optimal windows in oscillating domains

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### Abstract

Let  $\Omega \subset \mathbb{R}^n$  be a bounded domain with regular boundary. Let  $\alpha \in (0, 1)$ , and  $\Gamma \subset \partial\Omega$  a window such that  $|\Gamma|_{n-1} = \alpha|\Omega|_{n-1}$ . The optimal Sobolev trace constant is defined as

$$S(\Gamma) := \inf_{v \in W_{\Gamma}^{1,p}(\Omega)} \frac{\int_{\Omega} |\nabla v|^p + |v|^p dx}{\int_{\partial\Omega} |v|^p dS},$$

where  $W_{\Gamma}^{1,p}(\Omega)$  is the set of functions  $v \in W^{1,p}(\Omega)$  such that  $v|_{\Gamma} = 0$ .

In [Del Pezzo, F. Bonder, Neves, JDE (2011)], the authors study the following problem: minimize  $S(\Gamma)$  among all admissible windows, i.e.

$$S_{\alpha} = \inf_{\Gamma \in \Sigma_{\alpha}} S(\Gamma),$$

where  $\Sigma_{\alpha} = \{\Gamma \subset \partial\Omega : \text{are measurable for } dS \text{ and } |\Gamma|_{n-1} = \alpha|\partial\Omega|_{n-1}\}$ .

We make periodic perturbations of period  $\varepsilon$  and amplitude  $\varepsilon^a$  and we analyze the behavior of optimal windows  $\Gamma_{\varepsilon}^*$  when  $\varepsilon \rightarrow 0$ , we distinguish three cases: i.- Subcritical case: In this case the trace constant converges to zero. ii.- Supercritical case: In this case are convergence to the unperturbed problem. iii.- Critical case: In this case the amplitude compensates with the oscillations and this is reflected in the appearance of a weight term.

The results presented here are new and is for  $p = 2$ .

## A discrete Bernoulli free boundary problem and an application

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### Abstract

Let  $K \subset \{(0, x_2, \dots, x_n) \in \mathbb{R}^n\}$  be a smooth, compact, convex domain. Consider the Bernoulli problem of finding a convex and bounded domain  $\Omega$  contained in  $\mathbb{R}_+^n := \{x = (x_1, \dots, x_n); x_1 > 0\}$  such that  $\partial\Omega \supset K$  and a function  $u : \overline{\Omega} \rightarrow \mathbb{R}$  satisfying

$$\begin{cases} \Delta u = 0 \text{ in } \Omega, \\ u = 1 \text{ on } K, \\ u = 0 \text{ on } \partial\Omega \setminus K, \\ |\nabla u| = c \text{ on } \partial\Omega \cap \mathbb{R}_+^n. \end{cases}$$

where  $c$  is a positive constant. In this work we solve the above problem using a discrete Bernoulli free boundary problem.

## Approximate Controllability Of Semilinear Non–Autonomous In Hilbert Spaces

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### Abstract

In this paper we give a necessary and sufficient conditions for approximate controllability of a wide class of Semilinear Non-Autonomous Systems in Hilbert spaces. This is done by employing skew-product semi-flows technique. As an application we prove the approximate controllability of a broad class of non-autonomous semilinear reaction diffusion equations which includes the semilinear heat equation.