E' AND ITS RELATION WITH VECTOR-VALUED FUNCTIONS ON E

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Abstract. We study the relation between different spaces of vector-valued polynomials and analytic functions over dual-isomorphic Banach spaces. Under conditions of regularity on E and F, we show that the spaces of X-valued n-homogeneous polynomials and analytic functions of bounded type on E and F are isomorphic whenever X is a dual space. Also, we prove that many of the usual subspaces of polynomials and analytic functions on E and F are isomorphic without conditions on the involved spaces.

Introduction

Any Banach spaces E and F whose duals are isomorphic have, of course, the same linear forms. However, they do not necessarily have the same polynomials. Díaz and Dineen showed in [11] that if E' and F' are isomorphic and E' has the Schur property and the approximation property then, for any n, the spaces of scalar-valued n-homogeneous polynomials over E and F are isomorphic. In [5] and [22] it was shown that the result holds under conditions of regularity where the approximation and the Schur properties play no roll. In [22] the classical subspaces of polynomials were also studied and it was proved with no further conditions on E or F that those scalar-valued polynomials closely related to the structure of the dual spaces are isomorphic whenever E' and F' are isomorphic.

Our interest in these notes is to study the X-valued case of this problem: if E' and F' are isomorphic, are $P(^nE;X)$ and $P(^nF;X)$ (the spaces of X-valued n-homogeneous polynomials on E and F) isomorphic? We are also interested in how the different subspaces of polynomials are determined by E'.

One of the main difficulties to be dealt with in the vector-valued situation is that the natural generalization of the morphism constructed in [22] or [5] takes an X-valued polynomial on E to an X''-valued polynomial on F. Also, when we restrict the question to certain classes of polynomials, things are more complicated than in the scalar-valued case (specially for the integral polynomials).

The paper is organized as follows: In the first section we construct the morphism between the spaces of polynomials and give the general results. In the second, we deal with different classes of polynomials: finite type, nuclear, approximable, weakly continuous on bounded sets, regular, integral and extendible polynomials. We obtain without any assumption on the involved spaces the (isometric) isomorphism of each of the subspaces (except for that of extendible polynomials) whenever E' and F' are (isometrically) isomorphic. The third section is devoted to the study of different spaces of holomorphic functions on dual-isomorphic spaces.

Throughout, E, F, X and W are Banach spaces, E' is the dual space of E and $J_E : E \to E''$ is the natural embedding of E into its bidual. $P(^nE;X)$ and

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$L_s(^nE;X)$ denote, respectively, the spaces of continuous n-homogeneous polynomials and continuous symmetric n-linear mappings from E to X. If $P \in P(^nE;X)$ and A is its associated symmetric n-linear operator (i.e., $P(x) = A(x,\dots,x)$) we define some natural mappings which are associated to P and A:
Given $x \in E$, we denote A_x the $(n-1)$-linear operator given by

$$A_x(x_1,\dots,x_{n-1}) = A(x,x_1,\dots,x_{n-1})$$

and P_x the corresponding polynomial. Moreover, the mappings $T_A : E \to L_s(^{n-1}E;X)$ and $T_P : E \to P(^{n-1}E;X)$ are defined as $T_A(x) = A_x$ and $T_P(x) = P_x$ respectively.

We refer to [15] for general properties of polynomials, multilinear mappings and holomorphic functions on Banach spaces.

1. Construction of the morphism

For any linear map $s : E' \to F'$ we construct a morphism relating the spaces of polynomials on E and on F. In order to do this we define, via the Aron-Berner extension [1] and the construction in [22], a continuous linear map

$$\widetilde{s} : L_s(^nE;X) \to L(^nF;X'').$$

If Φ is a symmetric scalar-valued n-linear form on E, $\overline{\Phi}$ is its Aron-Berner extension and s' is the transpose of s, then $\widetilde{s}(\Phi)$ is defined for any $y_1,\dots,y_n \in F$ as follows (see [22, Lemma 1]):

$$\widetilde{s}(\Phi)(y_1,\dots,y_n) = \overline{\Phi}(s'(J_F(y_1)),\dots,s'(J_F(y_n))).$$

Now, we define for a symmetric n-linear function $A : E^n \to X$, $y_1,\dots,y_n \in F$ and $\varphi \in X'$

$$\widetilde{s}(A)(y_1,\dots,y_n)(\varphi) = \widetilde{s}(\varphi \circ A)(y_1,\dots,y_n)$$

Although $\widetilde{s}(A)$ it is not necessarily symmetric, the X''-valued n-homogeneous polynomial over F given by $\overline{s}(P)(y) = \widetilde{s}(A)(y,\dots,y)$, for all $y \in F$, is well defined. It is clear that if we take $s = J_{E'} : E' \to E'''$, the morphism $\overline{s}$ is the Aron-Berner extension. In this particular case we use the notation $\overline{P}$ and $\overline{A}$ for $\overline{s}(P)$ and $\widetilde{s}(A)$ respectively.
In what follows we often write y instead of $J_F(y)$. Also, we do not specify, unless it is necessary, the image of the function $\widetilde{s}(A)$, understanding that for any X-valued function A, $\widetilde{s}(A)$ is an X''-valued map.
The following results, that were obtained for the scalar-valued case in [22], remain true for the vector-valued case. Their proof are an immediate consequence of the extended definition of $\widetilde{s}$ and $\overline{s}$ and the scalar-valued results.

Lemma 1.1. *a) If $\overline{A}$ is symmetric, then $\overline{\widetilde{s}(A)} = \overline{A} \circ (s' \times \cdots \times s')$. Thus, $\overline{\widetilde{s}(A)}$ is also symmetric, and if P is the homogeneous polynomial associated to A, then $\overline{\overline{s}(P)} = \overline{P} \circ s'$.*
b) Suppose that $s : E' \to F'$ is an isomorphism, $P \in P(^nE;X)$ and A is its associated symmetric n-linear function. If $\overline{A}$ is symmetric then $(\overline{s^{-1}} \circ \overline{s})(P) = P$.

Note that in the second statement, $\widetilde{s}(A)$ is an element of $L_s(^nF;X'')$ and then we are considering the morphism $\widetilde{s^{-1}}$ acting on elements of $L_s(^nF;X'')$ and taking its values in $L(^nE;X^{iv})$. However, the result assures that $\widetilde{s^{-1}}(\widetilde{s}(A))$ belongs to $L_s(^nE;X)$, whenever $\overline{A}$ is symmetric. Since in symmetrically regular spaces the Aron-Berner extension of a symmetric multilinear mapping is also symmetric, we obtain the next theorem, the scalar-valued case of which was given in [5, 22].

Theorem 1.2. *If E and F are symmetrically Arens-regular, and E' and F' are (isometrically) isomorphic, then for any n, $\overline{s} : P(^nE; X) \to P(^nF; X'')$ is an (isometric) isomorphism with its image.*

In general, $\overline{s}(P)$ does not take its values in X, even when $(\overline{s^{-1}} \circ \overline{s})(P) = P$. For example, consider two non-isomorphic spaces E and F whose duals are isomorphic. The isomorphism $s : E' \to F'$ induces a mapping $\overline{s} : L(E; E) \to L(F; E'')$. If Id_E is the identity operator on E, then $\overline{s}(Id_E) = Id''_E \circ s' \circ J_F = Id_{E''} \circ s' \circ J_F$ and it takes its values in E if and only if $s'(F)$ is contained in E. But this would mean that s is the transpose of an isomorphism between E and F, leading us to a contradiction. However, if X is a dual space (say $X = W'$), something can be done. We define $\overline{s_W} : P(^nE; W') \to P(^nF; W')$ by

$$\overline{s_W}(P)(y)(w) = \overline{s}\left(w \circ P\right)(y) \qquad \text{for } y \in F \ , \ w \in W \ .$$

Note that $\overline{s}$ is applied to the scalar-valued polynomial $w \circ P = P(\cdot)(w)$. Therefore,

$$\overline{s}\left(w \circ P\right)(y) = \overline{w \circ P}(s'(y)) = \overline{P}(s'(y))(w) = \left(\overline{P} \circ s' \circ J_F(y)\right)(w).$$

This gives us an equivalent expression for $\overline{s_W}(P)$:

$$\overline{s_W}(P)(y) = \left(\overline{P} \circ s' \circ J_F(y)\right)|_W \ .$$

This second expression may seem more natural, but the first one matches better the proof of the following:

Theorem 1.3. *If E and F are symmetrically Arens-regular, and E' and F' are (isometrically) isomorphic, then for any n, $P(^nE; W')$ and $P(^nF; W')$ are (isometrically) isomorphic.*

Proof. Defining $\overline{s_W^{-1}} : P(^nF; W') \to P(^nE; W')$ in the obvious way, we have for $P \in P(^nE; W')$, $x \in E$ and $w \in W$,

$$\overline{s_W^{-1}} \circ \overline{s_W}(P)(x)(w) = \overline{s^{-1}}\left(w \circ \overline{s_W}(P)\right)(x) \ .$$

For $y \in F$, we have $w \circ \overline{s_W}(P)(y) = \overline{s_W}(P)(y)(w) = \overline{s}\left(w \circ P\right)(y)$ and by [22, Thm. 4],

$$\begin{aligned} \overline{s_W^{-1}} \circ \overline{s_W}(P)(x)(w) &= \overline{s^{-1}}\left(\overline{s}\left(w \circ P\right)\right)(x) \\ &= \left(w \circ P\right)(x) \\ &= P(x)(w). \end{aligned}$$

The reverse composition is analogous. Note that $\|\overline{s_W}(P)\| \leq \|P\| \, \|s\|^n$. Then, if s is an isometry the isometric result follows. □

In [16], P. Galindo, D. García, M. Maestre and J. Mujica give a construction similar to $\overline{s_W}$ using the sequence of operators introduced by Nicodemi in [23]. Although the main interest in [16] is the extension of multilinear operators, the proof of Theorem 9.3 can be adapted to obtain an analogous result to Theorem 1.3. We thank the referee for pointing out this fact. Though it is not obvious at first glance, the construction given in this paper coincides with the Nicodemi extension operators when applied to symmetric multilinear operators, which was proven in [21]. Therefore, following the proof of [16, Thm. 9.3] it is possible to obtain the same isomorphism as in Theorem 1.3. However, our expression for $\overline{s_W}$ will prove useful to study the usual subclasses of polynomials and analytic functions.

In the previous theorem W' can be replaced by any Banach space X which is complemented in its bidual. For the isometry, the projection $X'' \to X$ must be a norm one operator. Also, the hypothesis *E and F are simmetrically Arens-regular* can be replaced by *either E or F is Arens regular* (since if E' and F' are isomorphic and one of them is Arens regular, then so is the other).

2. $\overline{s}$ and some subspaces of polynomials

As it happens in the scalar valued case, it is natural to expect that those subspaces of polynomials which are closely related to E' are also preserved by $\overline{s}$. Since $\overline{s}$ ranges in $P(^nF; X'')$ one of the main tasks is to show that $\overline{s}(P)$ is X-valued for any P in the corresponding class. We will see that in many cases, an isomorphism between the dual spaces induces an isomorphism between the different subspaces of polynomials. Besides the classes of polynomials which are constructed by means of linear mappings (such as finite type, nuclear and approximable polynomials) this is true for weak-type, integral and regular polynomials, without any assumption on the spaces E, F or X.
On the other hand, we know that the weakly sequentially continuous polynomials are not, in general, preserved via the morphism $\overline{s}$ ([22]).

2.1. **Finite type, nuclear and approximable polynomials.** The formula $\overline{s}(P) = \overline{P} \circ s' \circ J_F$ shows that the subclasses of finite type, nuclear and approximable polynomials are preserved by $\overline{s}$.

Let P be an n-homogeneous polynomial of finite type, say $P = \sum_{j=1}^m \varphi_j^n w_j$, where $w_j \in X$ and $\varphi_j \in E'$, $j = 1, \ldots, m$. Then, $\overline{s}(P) = \sum_{j=1}^m s(\varphi_j)^n w_j$ and we have that $\overline{s}(P)$ is an X-valued finite type polynomial.
When P is an approximable n-homogeneous polynomial, there are n-homogeneous finite type polynomials $P_k \in P_f(^nE; X)$ approximating P in norm. The continuity of $\overline{s}$ and the completeness of X assure that $\overline{s}(P)$ is also an X-valued approximable polynomial.
Finally, recall that an n-homogeneous continuous polynomial P is said to be **nuclear** if there exists a representation of P such that $P = \sum_{j\geq 1} \varphi_j^n w_j$, where $(w_j)_{j\in\mathbb{N}} \subseteq X$ is a bounded sequence and $(\varphi_j)_{j\in\mathbb{N}} \subseteq E'$ is a sequence verifying $\sum_{j\geq 1} \|\varphi_j\|^n < \infty$.
The space of n-homogeneous nuclear polynomials, $P_N(^nE; X)$ is a Banach space endowed with the norm

$$\|P\|_N = \inf \left\{ \sum_{j\geq 1} \|\varphi_j\|^n \, \|w_j\| : \quad \sum_{j\geq 1} \varphi_j^n \, w_j \text{ a representation of } P \right\}.$$

Then, if $P = \sum_{j\geq 1} \varphi_j^n w_j$ is nuclear, $\overline{s}(P) = \sum_{j\geq 1} s(\varphi_j)^n w_j$ is also X-valued. On the other hand,

$$\begin{aligned} \|\overline{s}(P)\|_N &\leq \inf \left\{ \textstyle\sum_{j\geq 1} \|s(\varphi_j)\|^n \, \|w_j\| : \ \sum_{j\geq 1} \varphi_j^n \, w_j \text{ a representation of } P \right\} \\ &\leq \|s\|^n \, \|P\|_N. \end{aligned}$$

Thus, the mapping $\overline{s} : P_N(^nE; X) \to P_N(^nF; X)$ is a continuous operator.
Now, if $s : E' \to F'$ is an isomorphism, $\overline{s^{-1}} \circ \overline{s}(\varphi^n) = \left(s^{-1} \circ s(\varphi)\right)^n = \varphi^n$ for $\varphi \in E'$. This means that $\overline{s}$ is an isomorphism for the classes of finite type and nuclear polynomials. By density and continuity, this is also true for the space of approximable polynomials $P_a(^nE; X)$. The isomorphism is isometric if so is s.

2.2. **Weakly continuous polynomials on bounded sets.** Let $P_w({}^nE;X)$ be the space of polynomials which are weakly continuous on bounded sets. For a Banach space E such that E' has the approximation property, it was shown in [3] that $P_w({}^nE;X) \equiv P_a({}^nE,X)$. So if we consider a Banach space F whose dual is isomorphic to E', by the results of the previous section, we have $P_w({}^nE;X) \simeq P_w({}^nF;X)$. Also, it was shown in [22] that the isomorphism holds for the scalar-valued case, even when E' does not have the approximation property. The natural question is if the result is valid for the general case. The following lemma will be often used:

Lemma 2.1. *Let $A \in L_s({}^nE;X)$. If $T_A : E \to L_s({}^{n-1}E;X)$ is a weakly compact operator then, $\overline{A}$ is symmetric.*

Recall that polynomials that are weakly continuous on bounded sets are precisely those which are K-bounded, for some K a compact subset of E', (see [24], [4] for the scalar-valued case and [8] for the vector-valued case). For any bounded set K, the Aron-Berner extension of an X-valued K-bounded polynomial is an X''-valued K-bounded polynomial (see [7]). Moreover, the associated linear map of a w-continuous polynomial is a compact operator [3], and this assures that its Aron-Berner extension is in fact X-valued (as we will see in Proposition 2.5). As a consequence of this and with almost the same proof as in [22] we have:

Lemma 2.2. *Let $P \in P({}^nE;X)$ be K-bounded ($K \subseteq E'$), then $\overline{s}(P) \in P({}^nF;X)$ is $s(K)$-bounded ($s(K) \subseteq F'$) and*

$$\|\overline{s}(P)\|_{s(K)} \leq \|P\|_K.$$

Proposition 2.3. *If $s : E' \to F'$ is an (isometric) isomorphism, then*

$$\overline{s} : P_w({}^nE;X) \to P_w({}^nF;X)$$

is an (isometric) isomorphism.

2.3. **Regular Polynomials.** We say that $P : E \to X$ is a **regular** polynomial if its associated linear operator T_P is weakly compact. We denote $P_R({}^nE;X)$ the class of X-valued n-homogeneous regular polynomials on E endowed with the usual norm.

We describe the vector-valued version of the inclusion of $(P({}^kE))''$ into $P({}^kE'')$ studied in [2] and [20], which was introduced in [19]. First, define, for $z \in E''$, the mapping $e_z : P({}^kE;X) \to X''$ by $e_z(P) = \overline{P}(z)$. Let $\beta : (P({}^kE;X))'' \to P({}^kE'';X'')$ be given by

$$\beta(\Lambda)(z)(x') = \Lambda(x' \circ e_z),$$

for $\Lambda \in (P({}^nE;X))''$, $z \in E''$ and $x' \in X'$.

With the definitions and the diagram

$$E'' \xrightarrow{T_P''} (P({}^{n-1}E;X))'' \xrightarrow{\beta} P({}^{n-1}E'';X'')$$

we state next lemma:

Lemma 2.4. $T_{\overline{P}} = \beta \circ T_P''$.

Proof. Let $z_0, z \in E''$, for any $x' \in X'$ we have:

$$(\beta \circ T_P''(z_0))(z)(x') = T_P''(z_0)(x' \circ e_z) = z_0(T_P'(x' \circ e_z)). \tag{1}$$

Now, let x be in E. Following the notation in [26] we have

$$\begin{aligned} T'_P(x'\circ e_z)(x) &= x'\circ e_z(T_P(x)) = \overline{T_P(x)}(z)(x') \\ &= \overline{x'\circ T_P(x)}(z) = \overline{z}\circ\cdots\circ\overline{z}(x'\circ T_P(x)) \\ &= \overline{z}\circ\cdots\circ\overline{z}(x'\circ A_x) = \overline{z}\circ\cdots\circ\overline{z}(x'\circ A)(x). \end{aligned}$$

Since the last expresion is w^*-continuous in x, from (1) we have that $(\beta\circ T''_P(z_0))(z)(x') = z_0(\overline{z}\circ\cdots\circ\overline{z}(x'\circ A)) = \overline{A}(z_0, z, \ldots, z)(x') = (T_{\overline{P}}(z_0))(z)(x')$, as desired. □

The Aron-Berner extension preserves the class of regular polynomials in the following sense:

Proposition 2.5. *If* $P \in P_R(^nE; X)$ *then* $\overline{P} \in P_R(^nE''; X)$.

Proof. If P is a regular polynomial, then $\overline{P}$ is also regular as a consequence of Lemma 2.4. We see that $\overline{P}$ is X-valued by induction on n. Gantmacher's Theorem gives the result for $n = 1$. Now, suppose that the result holds for every $(n-1)$-homogeneous polynomial and let A be the symmetric n-linear function associated to P.
For $x_0 \in E$, let P_{x_0} be the $(n-1)$-homogeneous polynomial given by $P_{x_0}(x) = A(x_0, x, \ldots, x)$. We also define the operator $\varepsilon^1_{x_0} : P(^{n-1}E; X) \to P(^{n-2}E; X)$ as $\varepsilon^1_{x_0}(Q) = Q_{x_0}$. By the symmetry of A we have that $T_{P_{x_0}} = \varepsilon^1_{x_0}\circ T_P$. Since T_P is weakly compact so is $T_{P_{x_0}}$, which means that P_{x_0} is a regular polynomial. By inductive hypothesis $\overline{P_{x_0}}$ is X-valued. Since $\overline{P_{x_0}} = \left(\overline{P}\right)_{x_0}$ we can define, for $z \in E''$, the weakly compact mapping

$$\begin{aligned} E &\to X \\ x_0 &\mapsto \overline{A}(x_0, z, \ldots, z). \end{aligned}$$

The bitranspose of this operator is X-valued and in particular $\overline{P}(z) = \overline{A}(z, z, \ldots, z)$ belongs to X. □

We are ready to show the isomorphism result for regular polynomials.

Proposition 2.6. *If* $s : E' \to F'$ *is an (isometric) isomorphism, then*

$$\overline{s} : P_R(^nE; X) \to P_R(^nF; X)$$

is an (isometric) isomorphism.

Proof. We first show that $\overline{s}(P)$ is an element of $P_R(^nF; X)$. Let us see that $T_{\overline{s}(P)}$ is a w-compact operator. Consider the diagram

$$\begin{array}{ccc} F & \xrightarrow{T_{\overline{s}(P)}} & P(^{n-1}F; X) \\ s'\circ J_F \downarrow & & \uparrow \\ E'' & \xrightarrow{T_{\overline{P}}} & P(^{n-1}E''; X) \end{array}$$

If $y_0 \in F$, by Lemma 1.1, we have that $(T_{\overline{s}(P)}(y_0))(y) = (T_{\overline{P}}(s'\circ J_F)(y_0))((s'\circ J_F)(y))$. On the other hand, the morphism $Q \mapsto Q\circ(s'\circ J_F)$ is a continuous linear operator from $P(^{n-1}E''; X)$ to $P(^{n-1}F; X)$ that makes the diagram a commutative one, and $T_{\overline{s}(P)}$ is w-compact. The result follows from Lemmas 1.1 and 2.1. □

Before studying the class of integral polynomials, we present a generalization of the results for the two previous classes. Polynomials which are weakly continuous on bounded sets as well as regular polynomials can be considered in terms of some particular operator ideals: those of compact and weakly compact operators,

respectively. In this context, we can obtain (in a more abstract way) the results in Propositions 2.3 and 2.6. However, in our opinion the proofs given above are more constructive and some of the intermediate results have interest by themselves.
In order to proceed we use a factorization result given in [18]. We present a simplified version for our purposes.

Corollary 2.7. [18, Cor. 5] *Let $\mathcal{U}$ be a closed injective operator ideal. If $P \in P(^nE;X)$ the following are equivalent:*

i) The operator $T_P : E \to P(^{n-1}E;X)$ belongs to $\mathcal{U}$

ii) There exist a Banach space Y an operator $U \in \mathcal{U}(E;Y)$ and a polynomial $Q \in P(^nY;X)$ such that $P = Q \circ U$.

We denote by $P_{\mathcal{U}}(^nE;X)$ the subspace of $P(^nE;X)$ consisting of those polynomials satisfying *i)* or *ii)* of the previous Corollary. We can define, for $P \in P_{\mathcal{U}}(^nE;X)$, the norm $\|P\|_{\mathcal{U}} = \inf\{\|Q\|\,\|U\|^n\}$, where the infimum is taken over all factorizations of P with $U \in \mathcal{U}$.
Suppose that $\mathcal{U}$ is a closed injective operator ideal which is contained in $\mathcal{WC}o$ (the ideal of weakly continuous operators) verifying that for any $T \in \mathcal{U}$, T'' is also in $\mathcal{U}$. Then, if $P \in P_{\mathcal{U}}(^nE;X)$ we have that $\overline{P} \in P_{\mathcal{U}}(^nE'';X)$. Indeed, if P factors as in the Corollary then, $\overline{P} = \overline{Q} \circ U''$. Since U is weakly compact, $U''(E'') \subseteq Y$ and therefore $\overline{P}$ is X-valued. The fact that $U'' \in \mathcal{U}$ assures that $\overline{P} \in P_{\mathcal{U}}(^nE'';X)$. Moreover, $\|\overline{P}\|_{\mathcal{U}} \leq \|\overline{Q}\|\,\|\overline{U}\|^n = \|Q\|\,\|U\|^n$ and taking the infimum over all factorizations. we obtain $\|\overline{P}\|_{\mathcal{U}} \leq \|P\|_{\mathcal{U}}$. The injectiveness of $\mathcal{U}$ assures that the norms of $\overline{P}$ in $P_{\mathcal{U}}(^nE'';X)$ and $P_{\mathcal{U}}(^nE'';X'')$ coincide.
Note that $\mathcal{U} \subseteq \mathcal{WC}o$ implies that the Aron-Berner extension of the symmetric n-linear mapping associated to any $P \in P_{\mathcal{U}}$ is also symmetric. From these facts, Lemma 1.1 and a similar development as in the proof of Theorem 1.3 we can state the following:

Theorem 2.8. *Let $\mathcal{U} \subseteq \mathcal{WC}o$ be a closed injective operator ideal such that for any $T \in \mathcal{U}$, T'' is also in $\mathcal{U}$. If $s : E' \to F'$ is an (isometric) isomorphism, then*

$$\overline{s} : P_{\mathcal{U}}(^nE;X) \to P_{\mathcal{U}}(^nF;X)$$

is an (isometric) isomorphism.

If $\mathcal{U} = \mathcal{WC}o$, $P_{\mathcal{U}}(^nE;X)$ is precisely the space of regular polynomials, while for $\mathcal{U} = \mathcal{C}o$ (the ideal of compact operators), $P_{\mathcal{U}}(^nE;X)$ is the space of weakly continuous polynomials on bounded sets. In both cases, it can be seen that $\|P\|_{\mathcal{U}}$ coincides with $\|P\|$.

2.4. **Integral polynomials.** Recall that a polynomial $P \in P(^nE;X)$ is **integral** if there exists a regular X-valued Borel measure G, of bounded variation on $(B_{E'}, w^*)$ such that

$$P(x) = \int_{B_{E'}} \gamma(x)^n \; dG(\gamma)$$

for all $x \in E$. The space of n-homogeneous integral polynomials is denoted by $P_I(^nE,X)$ and the integral norm of a polynomial $P \in P_I(^nE,X)$ is defined as

$$\|P\|_I = \inf\{|G|(B_{E'})\},$$

where the infimum is taken over all measures G representing P.
It was proved in [9] that the Aron-Berner extension of an n-homogeneous scalar-valued integral polynomial P is also an integral polynomial and that the extension morphism is an isometry, i.e.: $\|\overline{P}\|_I = \|P\|_I$. We give a generalization to the vector-valued case of this result using very different technics. First, recall that if T :

$G \to X$ is an integral operator, then $T'' : G'' \to X''$ is an integral operator and $\|T\|_I = \|T''\|_I$ (this is a consequence of Corollaries 10 and 11 of [12, Chap. VIII, 2]). Since integral operators are weakly compact, T'' takes its values in X. Integral operators are not a regular ideal (i.e., an $X-$valued operator which is integral as an $X''-$ valued operator, need not be integral as an operator to X). However, for the bitranspose of an integral operator we have:

Proposition 2.9. *Let $T : G \to X$ be an integral operator. Then $T'' : G'' \to X$ is an integral operator and $\|T''\|_{L_I(G'',X)} = \|T''\|_{L_I(G'',X'')} = \|T\|_I$*

Proof. Since T is integral, given $\varepsilon > 0$, T admits a factorization

$$\begin{array}{ccc} G & \overset{T}{\to} & X \\ R\downarrow & & \uparrow S \\ C(K) & \overset{j}{\to} & L_1(\mu) \end{array}$$

where K is a compact topological space, μ is a regular Borel measure on K, j is the natural inclusion and $\|S\|\,\|j\|\,\|R\| \le \|T\|_I + \varepsilon$. The mapping j is integral with $\|j\|_I = \|j\|$. Thus, it is weakly compact and $j''(C(K)'') \subset L_1(\mu)$. If we see that $j'' : C(K)'' \to L_1(\mu)$ is integral, then we have that $T'' = S \circ j'' \circ R''$ is also integral (as an X-valued operator). We know that $j'' : C(K)'' \to L_1(\mu)''$ is integral and therefore absolutely 1-summing. This operator ideal is injective, so $j'' : C(K)'' \to L_1(\mu)$ is also absolutely 1-summing, with the same norm. Since $C(K)''$ has the metric extension property, then it is isometric to $C(L)$ for some L a compact topological space [10, I, 3.9]. Therefore, by [12, VI, 3, Thm. 12], $j'' : C(K)'' \to L_1(\mu)$ is integral and $\|j''\|_{L_I(C(K)'',L_1(\mu))} = \|j''\|_{\Pi_1(C(K)'',L_1(\mu))} = \|j''\|_{\Pi_1(C(K)'',L_1(\mu)'')} = \|j''\|_{L_I(C(K)'',L_1(\mu)'')} = \|j\|$. Now, $T'' : G'' \to X$ is integral and $\|T''\|_{L_I(G'',X)} \le \|S\| \circ \|j\| \circ \|R\| \le \|T\|_I + \varepsilon$, for any $\varepsilon > 0$. On the other hand, $\|T\|_I = \|T''\|_{L_I(G'',X'')} \le \|T''\|_{L_I(G'',X)}$ and this completes the proof. ☐

In [25] it is shown that the spaces $L_I({}^nE, X)$ and $L_I(\otimes_\epsilon^n E, X)$ are isometrically isomorphic. Next Proposition shows that analogous result for n-homogeneous polynomials holds. Note that it does not follow from the multilinear result, since the integral norm of a polynomial does not coincide with the integral norm of the associated symmetric multilinear operator (in fact, $\|A\|_I \le \|P\|_I \le \frac{n^n}{n!}\|A\|_I$).

Proposition 2.10. *The spaces $P_I({}^nE, X)$ and $L_I(\otimes_{s,\epsilon}^n E, X)$ are isometrically isomorphic.*

Proof. If $P \in P_I({}^nE, X)$, its linearization L_P belongs to $L_I(\otimes_{s,\epsilon}^n E, X)$ and $\|L_P\|_I \le \|P\|_I$ [8]. Suppose that $T \in L_I(\otimes_{s,\epsilon}^n E, X)$. Since $\otimes_{s,\epsilon}^n E$ is isometrically imbedded in $C(B_{E'})$, fixed $\varepsilon > 0$, T factors as in previous proposition:

$$\begin{array}{ccc} \otimes_{s,\epsilon}^n E & \overset{T}{\longrightarrow} & X \\ R\downarrow & & \uparrow S \\ C(B_{E'}) & \overset{j}{\longrightarrow} & L_1(\mu) \end{array}$$

The inclusion j is integral and then $S \circ j$ is a weakly compact operator on $C(B_{E'})$. By [12, Thm. VI.2.5], there exists a measure $G \in \mathcal{M}(C(B_{E'}); X)$ such that $S \circ j(f) = \int_{B_{E'}} f(\gamma)\ dG(\gamma)$ and $|G| = \|S \circ j\| \le \|T\|_I - \varepsilon$ (note that $\|R\| = 1$). Therefore, P, the polynomial associated to T, can be written

$$P(x) = \int_{B_{E'}} \gamma(x)^n\ dG(\gamma)\ .$$

This means that P is integral and $\|P\|_I \leq |G| \leq \|T\|_I - \varepsilon$ for any $\varepsilon > 0$ and the isometry follows. □

The next lemma is a consequence of [13, Thm. 2.2] and extends the fact that the bitranspose of an $X-$valued integral operator is also $X-$valued.

Lemma 2.11. *The Aron-Berner extension of an integral polynomial $P \in P_I({}^nE, X)$ is a polynomial $\overline{P}$ that takes values in X.*

Theorem 2.12. *If $P \in P_I({}^nE, X)$, then $\overline{P} \in P_I({}^nE''; X)$ and $\|\overline{P}\|_{P_I({}^nE'';X)} = \|P\|_I$.*

Proof. Take an integral polynomial $P : E \to X$. By Proposition 2.10, its linearization $L_P : \otimes^n_{s,\epsilon} E \to X$ is integral and has the same integral norm. Thus, by Proposition 2.9, L''_P is an X-valued integral operator (with the same norm). We have the diagram:

$$\begin{array}{ccc} (\otimes^n_{s,\epsilon} E)'' & \overset{L''_P}{\to} & X \\ & L & \\ i \uparrow & \nearrow & \\ \otimes^n_{s,\epsilon} E'' & & \end{array}$$

where the map $i : \otimes^n_{s,\epsilon} E'' \hookrightarrow (\otimes^n_{s,\epsilon} E)'' = P_I({}^nE)'$ is the inclusion via the identification given in [9]. That is, for an elementary tensor $z^{(n)} \in \otimes^n_{s,\epsilon} E''$, $i(z^{(n)})$ is the linear form on $P_I({}^nE)$ defined by $i(z^{(n)})(R) = \overline{R}(z)$, where $\overline{R} \in P_I({}^nE'')$ is the Aron-Berner extension of R.
Let $Q : E'' \to X$ be the polynomial

$$Q(z) = L(z \otimes \ldots \otimes z) = L''_P(i(z^{(n)})).$$

By Lemma 2.10, Q is integral and $\|Q\|_I \leq \|L''_P\|_I$. To show that $Q = \overline{P}$ take $x' \in X'$. Then,

$$x'(Q(z)) = x'\left(L''_P(i(z^n))\right) = i(z^n)(L'_P(x')) \ .$$

Note that $L'_P(x') \in (\otimes^n_{s,\epsilon} E)'$ is the polynomial $x' \circ P$. Then, for all $x' \in X'$

$$x'(Q(z)) = i(z^n)(x' \circ P) = \overline{x' \circ P}(z) = x'\left(\overline{P}(z)\right).$$

Thus, $\overline{P} : E'' \to X$ is integral and $\|\overline{P}\|_{P_I({}^nE'';X)} \leq \|L''_P\|_I = \|L_P\|_I = \|P\|_I$. The reverse inequality follows from $\|P\|_I \leq \|\overline{P}\|_{P_I({}^nE'';X)} \|J_E\| = \|\overline{P}\|_{P_I({}^nE'';X)}$. □

In order to prove that the vector-valued integral polynomials on E are determined by the dual space E' we prove first that every morphism $\overline{s}$ preserves that subclass.

Lemma 2.13. *If $P \in P({}^nE; X)$ is integral, then $\overline{s}(P) \in P({}^nF; X)$ is also integral, and*

$$\|\overline{s}(P)\|_I \leq \|s\|^n \|P\|_I.$$

Proof. As we have that $\overline{s}(P) = \overline{P} \circ s' \circ J_F$, the result is a consequence of the fact that integral polynomials form a right-ideal with continuous operators. Thus, by Theorem 2.12 we have

$$\|\overline{s}(P)\|_I = \|\overline{P} \circ s' \circ J_F\|_I \leq \|\overline{P}\|_I \|s\|^n = \|P\|_I \|s\|^n.$$

□

Now, we show that for any Banach spaces E, F with isomorphic dual spaces, the respective spaces of X-valued n-homogeneous integral polynomial are isomorphic.

Proposition 2.14. *If $s : E' \to F'$ is an (isometric) isomorphism, then*

$$\overline{s} : P_I(^nE, X) \to P_I(^nF; X)$$

is an (isometric) isomorphism.

Proof. In order to prove that $\overline{s^{-1}} \circ \overline{s}(P) = P$ when P is an integral polynomial its sufficient to prove that T_A is a weakly compact operator, where A is the n-linear symmetric function associated to P. The reverse composition is analogous. It is known that A is an integral multilinear mapping. To see that T_A is a weakly compact operator it is enough to see that $T_A : E \to L_I(^{n-1}E; X)$ is an integral operator.

It was proved in [25] that if $B : E_1 \times E_2 \to X$ is an integral bilinear mapping, then $B_1 : E_1 \to L_I(E_2; X)$ is an integral operator. Some modifications to the proof in [25] would lead to the desired result. However, we prefer to provide a shorter proof using the bilinear case.

Since A is integral, so is its linearization $L_A : \otimes_\varepsilon^n E \to X$. Identifying $\otimes_\varepsilon^n E$ with $E \otimes_\varepsilon \left(\otimes_\varepsilon^{n-1} E\right)$, we get a bilinear mapping $B : E \times \left(\otimes_\varepsilon^{n-1} E\right) \to X$ which is integral by the multilinear version of Proposition 2.10. By the bilinear case, $T_A = B_1 : E \to L_I\left(\otimes_\varepsilon^{n-1} E; X\right) = L_I\left(^{n-1}E; X\right)$ is an integral operator. □

2.5. **Extendible polynomials.** We say that $P : E \to X$ is an **extendible** polynomial if for any Banach space $Z \supseteq E$ there exists $Q : Z \to X$ extending P. The extendible norm of such a polynomial P can be defined as

$$\|P\|_e = \inf\{\|Q\|; \quad Q : C(B_{E'}) \to X \text{ extending } P\}.$$

It was mentioned in [22] that the spaces of scalar-valued extendible polynomials on E and F are (isometrically) isomorphic if so are E' and F'. We will give a proof of this fact in a more general context.

We have that if $P : E \to X$ is extendible, then its Aron-Berner extension $\overline{P} : E'' \to X''$ is also extendible, with $\left\|\overline{P}\right\|_e \leq \|P\|_e$. Also, $P \circ T$ is extendible for any continuous linear operator T on X with $\|P \circ T\|_e \leq \|P\|_e \|T\|^n$ [6, Thm. 3.4, Thm. 3.6]. However, the Aron-Berner extension of P needs not be X-valued. For instance, consider the identity map $id_{\ell_\infty} : \ell_\infty \to \ell_\infty$, which is extendible since ℓ_∞ is an injective space. Its Aron-Berner extension is the identity on $id_{(\ell_\infty)''}$ which is clearly not (ℓ_∞)-valued.

If X is a dual space, say $X = W'$, we consider the morphism $\overline{s_W}$ as in Theorem 1.3. Since $\overline{s_W}(P) = \rho \circ \overline{P} \circ s' \circ J_F$ (where $\rho : W''' \to W'$ is the restriction mapping), it is clear that $\overline{s_W}(P)$ is extendible with $\|\overline{s_W}(P)\|_e \leq \|P\|_e \|s\|^n$, whenever P is extendible.

To prove that an (isometric) isomorphism $s : E' \to F'$ induces an (isometric) isomorphism $\overline{s_W} : P_e(^nE, W') \to P_e(^nF, W')$ it is enough to show, by Lemma 1.1, that the Aron-Berner extension of the symmetric n-linear mapping A associated to each extendible polynomial P is also symmetric. Note that P can be extended to $C(B_{E'}, w^*)$, and therefore A factors through a symmetric n-linear mapping $B : C(B_{E'}) \times \cdots \times C(B_{E'}) \to W'$. $\overline{A}$ factors through $\overline{B}$, which is symmetric since $C(B_{E'})$ is symmetrically Arens-regular, and this assures the symmetry of $\overline{A}$. We have obtained the following:

Proposition 2.15. *If E' and F' are (isometrically) isomorphic, then for any Banach space W, the spaces $P_e(^nE, W')$ and $P_e(^nF; W')$ are (isometrically) isomorphic.*

2.6. **One Example.** It was shown in [22] that the subclass of weakly sequentially continuous polynomials is not preserved, in general, by $\overline{s}$. With the following example we show that the class could be preserved under certain conditions.

Example 2.16. *Let E be a separable Banach space such that $E \not\supseteq \ell_1$. If F' is isomorphic to E', then the spaces $P_{wsc}(^nE)$ and $P_{wsc}(^nF)$ are isomorphic.*

Proof. Recall that by a result of Odell and Rosenthal, a separable Banach space contains ℓ_1 if and only if the cardinality of its bidual is greater than c. Since $E \not\supseteq \ell_1$ and E' is isomorphic to F', F cannot contain ℓ_1. Therefore, $P_{wsc}(^nE) = P_w(^nE)$ and $P_{wsc}(^nF) = P_w(^nF)$ (see [3] Prop. 2.12) and the result follows from Proposition 2.3. □

Note that we need only to impose conditions on one of the spaces.

3. Holomorphic functions

In this section we investigate the relation between the different Fréchet algebras or spaces of holomorphic functions on Banach spaces whose duals are isomorphic. Most of the work has already been done in the previous sections, where the behaviour of the mapping $\overline{s}$ (or $\overline{s_W}$) on different spaces of polynomials was studied. Recall that if U is an open subset of E, $H_b(U, X)$ is the space of X-valued holomorphic functions of bounded type on U, that is, the functions which are bounded on subsets $V \subset U$ which are bounded and bounded away from the boundary of U. $H_b(U, X)$ is a Fréchet space with the family of seminorms

$$p_V(f) = \sup_V \|f\|.$$

On the other hand, $H^\infty(U, X)$ denotes the space of bounded holomorphic functions from U to X. This is a Banach space when equipped with the sup norm. If X is an algebra, $H_b(U, X)$ and $H^\infty(U, X)$ are, respectively, Fréchet and Banach algebras.

In order to derive conclusions for analytic functions from the results obtained for polynomials, we need the following:

Lemma 3.1. *Let $U \subset E$ be an open subset containing 0 and $f : U \to X$ an analytic function whose Taylor series expansion at 0, $f(x) = \sum_{k\geq 0} P_k(x)$, converges uniformly on rB_E. Then,*
a) $\overline{f} \circ s' \circ J_F = \sum_{k\geq 0} \overline{s}(P_k)$ uniformly on $\frac{r}{\|s\|} B_F$.
b) If $X = W'$, $\overline{f} \circ s' \circ J_F(y) \mid_W = \sum_{k\geq 0} \overline{s_W}(P_k)(y)$ uniformly for $\|y\| \leq \frac{r}{\|s\|}$.

Proof. a) Since $\overline{f} = \sum_{k\geq 0} \overline{P_k}$ converges uniformly on $rB_{E''}$ [1], we have that $\overline{f} \circ s' \circ J_F(y) = \sum_{k\geq 0} \overline{P_k} \circ s' \circ J_F(y) = \sum_{k\geq 0} \overline{s}(P_k)$ (and the series converges uniformly), whenever $\|s' \circ J_F(y)\| \leq r$. In particular, this holds if $\|y\| \leq \frac{r}{\|s\|}$.
b) The statement follows applying the restriction mapping $\rho : W''' \to W'$ to the equality obtained in *a)*. □

Suppose E and F are symmetrically regular and $X = W'$ is a dual space. If $f \in H_b(E, W')$, we can define $\overline{s_W}(f) \in H_b(F, W')$ by $\overline{s_W}(f)(y) = \overline{f} \circ s' \circ J_F(y) \mid_W$, which coincides with $\sum_{k\geq 0} \overline{s_W}(P_k)(y)$. To see that $\overline{s_W}(f)$ is a bounded type holomorphic function, observe that if f has infinite radius of uniform convergence, by Lemma 3.1 *(b)*, $\overline{s_W}(f)$ has also infinite radius of uniform convergence. Theorem 1.3 (applied to each polynomial on the expansion of f) and the fact that the Aron-Berner extension is multiplicative, give the first statement of the following proposition.

Proposition 3.2. *Let E and F are symmetrically Arens-regular whose duals are isomorphic. Then:*
a) $H_b(E; W')$ and $H_b(F; W')$ are isomorphic Fréchet spaces.
If the isomorphism between E' and F' is isometric, then
b) $H_b(B_E; W')$ and $H_b(B_F; W')$ are isomorphic Fréchet spaces.
c) $H^\infty(B_E; W')$ and $H^\infty(B_F; W')$ are isometrically isomorphic Banach spaces.
If W' is a Banach algebra (in particular, if W' is the scalar field), $\overline{s_W}$ is an isomorphism of Fréchet/Banach algebras.

Proof. To prove *b)*, we have to show that if $f \in H_b(B_E; W')$, then $\overline{s_W}(f) \in H_b(B_F; W')$. But this follows from the fact that $s' \circ J_F(rB_F)$ is contained in rB_E, since s is an isometry. The result is now a consequence of Lemma 3.1 *(b)* and Theorem 1.3. The proof of *c)* is analogous. □

The scalar-valued case of the first statement is in [5]. It is worthwhile to note that s needs be an isometric isomorphism for $\overline{s}$ to be an isomorphism in b) and c) in the previous proposition, even for the scalar-valued case. The same holds for Propositions 3.3 and 3.4.

As we have seen in the first section, the assumption that X be a dual space cannot be omitted, unless restrictions are made on the polynomials which are involved. Naturally, the same occurs with analytic functions. We need not make assumptions on X for those classes of analytic functions related to spaces of polynomials where $\overline{s}$ has a good behaviour. We point this out with two examples: holomorphic functions which are uniformly weakly continuous on bounded sets and boundedly integral functions.
Let $H_{wu}(E; X)$ be the space of holomorphic function which are uniformly weakly continuous on bounded sets. Analogously, $H_{wu}(B_E; X)$ consists of holomorphic functions on B_E which are uniformly weakly continuous on rB_E for $r < 1$. A function $f : E \to X$ belongs to $H_{wu}(E; X)$ if and only if it has an infinite radius of uniform convergence (at 0) and every polynomial in its Taylor series expansion is weakly continuous on bounded sets (for $H_{wu}(B_E; X)$, the radius must be at least 1). Therefore, from Proposition 2.3 and Lemma 3.1 *(a)* we have:

Proposition 3.3. *a) If E' and F' are isomorphic, then $H_{wu}(E; X)$ and $H_{wu}(F; X)$ are isomorphic Fréchet spaces.*
b) If E' and F' are isometrically isomorphic, $H_{wu}(B_E; X)$ and $H_{wu}(B_F; X)$ are isomorphic Fréchet spaces.
If X is a Banach algebra (in particular, if X is the scalar field), $\overline{s}$ is an isomorphism of Fréchet algebras.

Now we study the boundedly integral functions introduced for the scalar-valued case in [14]. A function $f : B_E \to X$ is integral if there exists an X-valued measure G on $(B_{E'}, w^*)$ such that

$$f(x) = \int_{B_{E'}} \frac{1}{1 - \gamma(x)} dG(\gamma) \ . \tag{2}$$

Integral functions are holomorphic and each polynomial in its Taylor series expansion is integral.bb
A function $f : B_E \to X$ is boundedly integral if $f_r = f(r\cdot)$ is integral for any $0 < r < 1$. Proposition 11 in [14] (which readily extends to the vector-valued case) states that a holomorphic function $f = \sum_k P_k$ is boundedly integral ($f \in$

$H_{bI}(B_E, X))$ if and only if each P_k is an integral polynomial and $r_I := \frac{1}{\limsup \|P\|_I}$ is at least 1.
On the other hand, a function $f : E \to X$ is boundedly integral if $f\,|_{nB_E}$ is integral in the sense of expression (2), with a measure defined on $\frac{1}{n}B_{E'}$, for all $n \in I\!N$. It can be seen that $f = \sum_k P_k$ is boundedly integral on E if and only if each P_k is an integral polynomial and $r_I = +\infty$.
As a consequence of Proposition 2.14 and Lemma 3.1 we have:

Proposition 3.4. *a) If E' and F' are isomorphic, $H_{bI}(E;X)$ and $H_{bI}(F;X)$ are isomorphic Fréchet spaces.*
b) If E' and F' are isometrically isomorphic, $H_{bI}(B_E;X)$ and $H_{bI}(B_F;X)$ are isomorphic Fréchet spaces.

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