White noise perturbation of a reaction-diffusion equation with explosions

Pablo Groisman

Universidad de Buenos Aires

Joint work with

J. Fernández Bonder, UBA

5to Encuentro Regional de Probabilidad y Estadística Matemática
Noviembre 2008

Outline

- Deterministic reaction-diffusion equations.
- Semilinear Stochastic Partial Differential Equations.
- Explosions in the deterministic case.
- Explosions in the perturbed equation.
- Numerical approximations.
- Some simulations.
- Future work

(Deterministic) Reaction-Diffusion equations

$$\begin{cases} u_t(x,t) = \Delta u(x,t) + f(u(x,t)), & x \in U, 0 < t < T \\ u(x,t) = 0, & x \in \partial U, 0 < t < T \\ u(x,0) = u_0(x), & x \in U \end{cases}$$

(Deterministic) Reaction-Diffusion equations

$$\begin{cases} u_t(x,t) = \Delta u(x,t) + f(u(x,t)), & x \in U, 0 < t < T \\ u(x,t) = 0, & x \in \partial U, 0 < t < T \\ u(x,0) = u_0(x), & x \in U \end{cases}$$

Useful to model:

- Chemical reactions,
- Chemotaxis in biological systems,
- Population dynamics,
- etc.

Explosions

If f is globally Lipschitz, every solution is global, i.e. solutions are defined for every positive time.

Explosions

If f is globally Lipschitz, every solution is global, i.e. solutions are defined for every positive time.

If f is just locally Lipschitz, solutions could be defined just locally in time.

Blow-up

There exists a finite time T such that $u(x,t)$ is defined for every $0 \leq t < T$, but

$$\lim_{t \nearrow T} \|u(\cdot,t)\|_{L^\infty(U)} = +\infty$$

Blow-up

There exists a finite time T such that $u(x,t)$ is defined for every $0 \leq t < T$, but

$$\lim_{t \nearrow T} \|u(\cdot,t)\|_{L^\infty(U)} = +\infty$$

Example: The ODE

$$\dot{u}(t) = u^2(t), u(0) = 1$$

The solution

$$u(t) = \tfrac{1}{1-t},$$

blows up at time $T = 1$

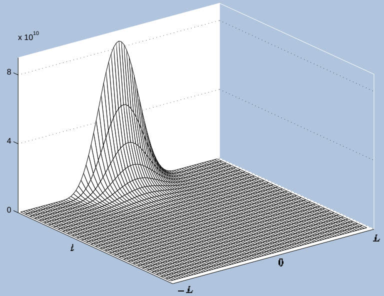
In PDEs

$$u_t = \Delta u + f(u) \quad \text{in } U \times (0, T)$$

If f verifies

- $f \geq 0$
- f convex
- $\int^{\infty} \frac{1}{f} < \infty$

Then, there exist initial data u_0 such that u blows-up in finite time.



Remark:

- Solutions are classical up to time T.
- There is no reasonable way to extend the solution after time T (Complete blow-up).

Stochastic Partial Differential Equations

$$u_t = \Delta u + f(u) + \sigma \dot{W}(x,t) \quad \text{in } U \times (0,T)$$

$\dot{W}(x,t)$ is two parameter white-noise

Stochastic Partial Differential Equations

$$u_t = \Delta u + f(u) + \sigma \dot{W}(x,t) \quad \text{in } U \times (0,T)$$

$\dot{W}(x,t)$ is two parameter white-noise

Heuristically

- $\dot{W}(x,t) = \frac{\partial^2}{\partial x \partial t} W(x,t)$, ($W$ =Brownian sheet).
- $\dot{W}$ is a zero mean Gaussian field with $\mathrm{Cov}(\dot{W}(x,t), \dot{W}(y,s)) = \delta_0((x,t)-(y,s))$.

Stochastic Partial Differential Equations

$$u_t = \Delta u + f(u) + \sigma \dot{W}(x,t) \quad \text{in } U \times (0,T)$$

$\dot{W}(x,t)$ is two parameter white-noise

Heuristically

- $\dot{W}(x,t) = \frac{\partial^2}{\partial x \partial t} W(x,t)$, ($W$ =Brownian sheet).
- $\dot{W}$ is a zero mean Gaussian field with $\mathrm{Cov}(\dot{W}(x,t), \dot{W}(y,s)) = \delta_0((x,t)-(y,s))$.

Rigourously

$$\{W(A) \colon A \text{ a Borel set of } U \times \mathbb{R}_+\}$$

is a centered Gaussian random field with covariance given by

$$\mathbb{E}[W(A)W(B)] = |A \cap B|$$

Stochastic Partial Differential Equations

$$u_t = \Delta u + f(u) + \sigma \dot{W}(x,t) \quad \text{in } U \times (0,T)$$

$\dot{W}(x,t)$ is two parameter white-noise

Heuristically

- $\dot{W}(x,t) = \frac{\partial^2}{\partial x \partial t} W(x,t)$, ($W$ =Brownian sheet).
- $\dot{W}$ is a zero mean Gaussian field with $\mathrm{Cov}(\dot{W}(x,t), \dot{W}(y,s)) = \delta_0((x,t)-(y,s))$.

Rigourously

$$\{W(A) \colon A \text{ a Borel set of } U \times \mathbb{R}_+\}$$

is a centered Gaussian random field with covariance given by

$$\mathbb{E}[W(A)W(B)] = |A \cap B|$$

We consider the filtration

$$\mathcal{F}_t = \sigma\{W(A) \colon A \text{ a Borel set of } U \times [0,t]\}$$

Stochastic Partial Differential Equations

For $\varphi(x) = \mathbf{1}_A$ we define

$$\int\int \varphi(x) dW(x,t) := W(A)$$

and we extend this definition for $\varphi \in C^2(U) \cap C_0(\bar{U})$

Weak solutions to the SPDE

or what is a solution to $u_t = \Delta u + f(u) + \sigma \dot{W}$*?*

Weak solutions to the SPDE

or what is a solution to $u_t = \Delta u + f(u) + \sigma \dot{W}$?

Let $\varphi \in C^2(U) \cap C_0(\bar{U})$, then formally, for every $0 \leq t \leq T$

$$\int_U u(x,t)\varphi(x)\,dx - \int_U u_0(x)\varphi(x)\,dx =$$

$$\int_0^t \int_U \Delta u \varphi\,dxds + \int_0^t \int_U f(u)\varphi\,dxdt + \int_0^t \int_U \sigma\varphi \dot{W}(x,s)\,dxds$$

Weak solutions to the SPDE

or what is a solution to $u_t = \Delta u + f(u) + \sigma \dot{W}$?

Let $\varphi \in C^2(U) \cap C_0(\bar{U})$, then formally, for every $0 \leq t \leq T$

$$\int_U u(x,t)\varphi(x)\,dx - \int_U u_0(x)\varphi(x)\,dx =$$
$$\int_0^t \int_U \Delta u \varphi \, dxds + \int_0^t \int_U f(u)\varphi \, dxdt + \int_0^t \int_U \sigma\varphi \dot{W}(x,s)\,dxds$$

$$\int_U u(x,t)\varphi(x)\,dx - \int_U u_0(x)\varphi(x)\,dx =$$
$$\int_0^t \int_U u\Delta\varphi \, dxds + \int_0^t \int_U f(u)\varphi \, dxdt + \int_0^t \int_U \sigma\varphi dW(x,s)$$

Solutions to the SPDE

Walsh, 1986, Gyöngy and Pardoux, 1993, Buckdahn and Pardoux, 1989:
This formulation is equivalent to the integral formulation in terms of the fundamental solution of the Heat Equation.

$$u(x,t) = \int_U G(x-y,t)u_0(y)\,dy + \int_0^t \int_U G(x-y,t-s)f(u(s,y))\,dsdy + \int_0^t \int_U G(x-y,t-s)\,dW(y,s)$$

$$G(x,t) = \text{Fundamental solution of the heat equation}$$
$$= \frac{1}{(2\pi t)^{d/2}} \exp\left(-\frac{x^2}{2t}\right) \text{ if } U = \mathbb{R}^d.$$

Existence and uniqueness

- If f is globally Lipschitz, such an u exists (and is unique) for every $t \geq 0$ but just for dimension one.

Existence and uniqueness

- If f is globally Lipschitz, such an u exists (and is unique) for every $t \geq 0$ but just for dimension one.
- In dimensions higher than one u is not a function and hence, in fact, do not exist a solution to our problem as a random function (There exist a solution for $f = 0$ but as a distribution).

Existence and uniqueness

- If f is globally Lipschitz, such an u exists (and is unique) for every $t \geq 0$ but just for dimension one.
- In dimensions higher than one u is not a function and hence, in fact, do not exist a solution to our problem as a random function (There exist a solution for $f = 0$ but as a distribution).
- So we consider just the one-dimensional case $U = (0, 1)$, where u is a random function.

The locally Lipschitz case

Solutions up to an explosion time

If f is just locally Lipschitz, consider for each $n \in \mathbb{N}$ the globally Lipschitz function

$$f_n(x) = f(-n)\mathbf{1}_{(-\infty,-n]} + f(x)\mathbf{1}_{(-n,n)} + f(n)\mathbf{1}_{[n,+\infty)}$$

The locally Lipschitz case

Solutions up to an explosion time

If f is just locally Lipschitz, consider for each $n \in \mathbb{N}$ the globally Lipschitz function

$$f_n(x) = f(-n)\mathbf{1}_{(-\infty,-n]} + f(x)\mathbf{1}_{(-n,n)} + f(n)\mathbf{1}_{[n,+\infty)}$$

and u^n, the unique solution to

$$u^n_t = \Delta u^n + f_n(u) + \sigma \dot{W}.$$

The locally Lipschitz case

Solutions up to an explosion time

If f is just locally Lipschitz, consider for each $n \in \mathbb{N}$ the globally Lipschitz function

$$f_n(x) = f(-n)\mathbf{1}_{(-\infty,-n]} + f(x)\mathbf{1}_{(-n,n)} + f(n)\mathbf{1}_{[n,+\infty)}$$

and u^n, the unique solution to

$$u^n_t = \Delta u^n + f_n(u) + \sigma \dot{W}.$$

- $T_n = \inf\{t > 0 \colon \|u^n(\cdot, t)\|_\infty \geq n\}$

The locally Lipschitz case

Solutions up to an explosion time

If f is just locally Lipschitz, consider for each $n \in \mathbb{N}$ the globally Lipschitz function

$$f_n(x) = f(-n)\mathbf{1}_{(-\infty,-n]} + f(x)\mathbf{1}_{(-n,n)} + f(n)\mathbf{1}_{[n,+\infty)}$$

and u^n, the unique solution to

$$u^n_t = \Delta u^n + f_n(u) + \sigma \dot{W}.$$

- $T_n = \inf\{t > 0 \colon \|u^n(\cdot,t)\|_\infty \geq n\}$
- $u^{n+1}\mathbf{1}_{\{t<T_n\}} = u^n\mathbf{1}_{\{t<T_n\}}$ a.s.

The locally Lipschitz case

Solutions up to an explosion time

If f is just locally Lipschitz, consider for each $n \in \mathbb{N}$ the globally Lipschitz function

$$f_n(x) = f(-n)\mathbf{1}_{(-\infty,-n]} + f(x)\mathbf{1}_{(-n,n)} + f(n)\mathbf{1}_{[n,+\infty)}$$

and u^n, the unique solution to

$$u^n_t = \Delta u^n + f_n(u) + \sigma \dot{W}.$$

- $T_n = \inf\{t > 0 \colon \|u^n(\cdot,t)\|_\infty \geq n\}$
- $u^{n+1}\mathbf{1}_{\{t<T_n\}} = u^n\mathbf{1}_{\{t<T_n\}}$ a.s.
- $T_{n+1} \geq T_n$ (hence $T = \lim_{n\to\infty} T_n$ exists).

The locally Lipschitz case

Solutions up to an explosion time

If f is just locally Lipschitz, consider for each $n \in \mathbb{N}$ the globally Lipschitz function

$$f_n(x) = f(-n)\mathbf{1}_{(-\infty,-n]} + f(x)\mathbf{1}_{(-n,n)} + f(n)\mathbf{1}_{[n,+\infty)}$$

and u^n, the unique solution to

$$u^n_t = \Delta u^n + f_n(u) + \sigma \dot{W}.$$

- $T_n = \inf\{t > 0 \colon \|u^n(\cdot,t)\|_\infty \geq n\}$
- $u^{n+1}\mathbf{1}_{\{t<T_n\}} = u^n\mathbf{1}_{\{t<T_n\}}$ a.s.
- $T_{n+1} \geq T_n$ (hence $T = \lim_{n\to\infty} T_n$ exists).
- Also $u(x,t) = \lim u^n(x,t)$ exists for $t < T$ and verifies

The locally Lipschitz case

Solutions up to an explosion time

If f is just locally Lipschitz, consider for each $n \in \mathbb{N}$ the globally Lipschitz function

$$f_n(x) = f(-n)\mathbf{1}_{(-\infty,-n]} + f(x)\mathbf{1}_{(-n,n)} + f(n)\mathbf{1}_{[n,+\infty)}$$

and u^n, the unique solution to

$$u_t^n = \Delta u^n + f_n(u) + \sigma \dot{W}.$$

- $T_n = \inf\{t > 0 \colon \|u^n(\cdot,t)\|_\infty \geq n\}$
- $u^{n+1}\mathbf{1}_{\{t<T_n\}} = u^n\mathbf{1}_{\{t<T_n\}}$ a.s.
- $T_{n+1} \geq T_n$ (hence $T = \lim_{n\to\infty} T_n$ exists).
- Also $u(x,t) = \lim u^n(x,t)$ exists for $t < T$ and verifies

$$\int_0^1 u(x,t\wedge T)\varphi(x)\,dx - \int_0^1 u_0(x)\varphi(x)\,dx =$$

$$\int_0^{t\wedge T}\int_0^1 u\varphi_{xx}\,dx\,ds + \int_0^{t\wedge T}\int_0^1 f(u)\varphi\,dx\,ds + \int_0^{t\wedge T}\int_0^1 \varphi\,dW.$$

Remark: If $T(\omega) < \infty$ then

$$\lim_{t \nearrow T(\omega)} \|u(\cdot, t, \omega)\|_\infty = \infty.$$

Explosions

An important issue is to determine in terms of u_0, f and ω whether $T(\omega) < \infty$ or not.

Explosions

An important issue is to determine in terms of u_0, f and ω whether $T(\omega) < \infty$ or not.

Deterministic case (σ =0).

Several criteria:

Giga-Kohn, Cortazar-Del Pino-Elgueta, etc.

- Energy methods:

$$\Phi(u)(t) = \frac{1}{2}\int_0^1 u_x^2(x,t)\,dx - \int_0^1 F(u(x,t))\,dx \quad (F' = f).$$

  u blows up at time $T \iff \lim_{t \nearrow T} \Phi(u)(t) = -\infty \iff \Phi(u)(t_0) < 0$ for some $t_0 < T$
- First eigenfunction method.
- Stationary solutions: Let v the unique positive solution of $v_{xx} + f(v) = 0$

Deterministic case (σ =0).

Several criteria:

Giga-Kohn, Cortazar-Del Pino-Elgueta, etc.

- Energy methods:

$$\Phi(u)(t) = \frac{1}{2}\int_0^1 u_x^2(x,t)\,dx - \int_0^1 F(u(x,t))\,dx \quad (F' = f).$$

  u blows up at time $T \iff \lim_{t\nearrow T}\Phi(u)(t) = -\infty \iff \Phi(u)(t_0) < 0$ for some $t_0 < T$

- First eigenfunction method.
- Stationary solutions: Let v the unique positive solution of $v_{xx} + f(v) = 0$
  - If $u_0 \leq v$ then $u(x,t) \to 0$ as $t \to +\infty$.

Deterministic case (σ =0).

Several criteria:

Giga-Kohn, Cortazar-Del Pino-Elgueta, etc.

- Energy methods:

$$\Phi(u)(t) = \frac{1}{2}\int_0^1 u_x^2(x,t)\,dx - \int_0^1 F(u(x,t))\,dx \quad (F' = f).$$

  u blows up at time $T \iff \lim_{t \nearrow T} \Phi(u)(t) = -\infty \iff \Phi(u)(t_0) < 0$ for some $t_0 < T$

- First eigenfunction method.
- Stationary solutions: Let v the unique positive solution of $v_{xx} + f(v) = 0$
  - If $u_0 \leq v$ then $u(x,t) \to 0$ as $t \to +\infty$.
  - If $u_0 \geq v$ then u blows up in finite time.

Deterministic case (σ =0).

Several criteria:

Giga-Kohn, Cortazar-Del Pino-Elgueta, etc.

- Energy methods:

$$\Phi(u)(t) = \frac{1}{2}\int_0^1 u_x^2(x,t)\,dx - \int_0^1 F(u(x,t))\,dx \quad (F' = f).$$

  u blows up at time $T \iff \lim_{t \nearrow T} \Phi(u)(t) = -\infty \iff \Phi(u)(t_0) < 0$ for some $t_0 < T$
- First eigenfunction method.
- Stationary solutions: Let v the unique positive solution of $v_{xx} + f(v) = 0$
  - If $u_0 \leq v$ then $u(x,t) \to 0$ as $t \to +\infty$.
  - If $u_0 \geq v$ then u blows up in finite time.
- This criteria does not decide for other values of u_0

Stochastic case

Theorem. Let f be a nonnegative, convex locally Lipschitz function such that

$$\int^{\infty} \frac{1}{f} < \infty.$$

Then, for every initial datum u_0 and for every positive σ, u blows-up in a (random) time T with

$$\mathbb{P}^{u_0}(T < \infty) = 1.$$

Stochastic case

Theorem. Let f be a nonnegative, convex locally Lipschitz function such that

$$\int^{\infty} \frac{1}{f} < \infty.$$

Then, for every initial datum u_0 and for every positive σ, u blows-up in a (random) time T with

$$\mathbb{P}^{u_0}(T < \infty) = 1.$$

Very different to the behavior of the deterministic problem $\sigma = 0$

Proof. Let $\varphi(x) = 2\sin(\pi x)$, which verifies

$$\begin{aligned}
&\varphi''(x) = -\pi^2\varphi(x)\\
&\|\varphi\|_{L^2(0,1)} = 1\\
&\varphi(x) > 0, \quad \forall\ x \in (0,1)
\end{aligned}$$

Proof. Let $\varphi(x) = 2\sin(\pi x)$, which verifies

$$\varphi''(x) = -\pi^2\varphi(x)$$
$$\|\varphi\|_{L^2(0,1)} = 1$$
$$\varphi(x) > 0, \quad \forall \; x \in (0,1)$$

Let

$$\Phi(t) := \int_0^1 \varphi(x)u(x,t)\,dx.$$

Proof. Let $\varphi(x) = 2\sin(\pi x)$, which verifies

$$\begin{aligned} &\varphi''(x) = -\pi^2\varphi(x) \\ &\|\varphi\|_{L^2(0,1)} = 1 \\ &\varphi(x) > 0, \quad \forall\ x \in (0,1) \end{aligned}$$

Let

$$\Phi(t) := \int_0^1 \varphi(x)u(x,t)\,dx.$$

We have

$$\Phi(t) = -\pi^2\int_0^t \Phi(s)\,ds + \int_0^t\int_0^1 \varphi(x)f(u(x,s))\,dxds + \sigma\int_0^t\int_0^1 \varphi(x)\,dW.$$

Proof. Let $\varphi(x) = 2\sin(\pi x)$, which verifies

$$\begin{aligned}&\varphi''(x) = -\pi^2\varphi(x)\\ &\|\varphi\|_{L^2(0,1)} = 1\\ &\varphi(x) > 0, \quad \forall\ x \in (0,1)\end{aligned}$$

Let

$$\Phi(t) := \int_0^1 \varphi(x)u(x,t)\,dx.$$

We have

$$\Phi(t) = -\pi^2\int_0^t \Phi(s)\,ds + \int_0^t\int_0^1 \varphi(x)f(u(x,s))\,dxds + \sigma\int_0^t\int_0^1 \varphi(x)\,dW.$$

As f is convex, by Jensen's inequality, we get

$$\int_0^1 \varphi(x)f(u(x,s))\,dx \geq \alpha f\Big(\int_0^1 \varphi(x)u(x,s)\,dx\Big) = \alpha f(\Phi(s)).$$

Moreover, since φ is a positive function with L^2-norm equal to 1

$$B(t) := \int_0^t \int_0^1 \varphi(x)\, dW(x,s),$$

is a Brownian motion.

Moreover, since φ is a positive function with L^2-norm equal to 1

$$B(t) := \int_0^t \int_0^1 \varphi(x)\, dW(x,s),$$

is a Brownian motion.
Then Φ verifies the one dimensional stochastic differential inequality

$$d\Phi(t) \geq \big(-\pi^2 \Phi(t) + \alpha f(\Phi(t)) \big)\, dt + \sigma dB(t).$$

Moreover, since φ is a positive function with L^2-norm equal to 1

$$B(t) := \int_0^t \int_0^1 \varphi(x)\, dW(x,s),$$

is a Brownian motion.

Then Φ verifies the one dimensional stochastic differential inequality

$$d\Phi(t) \geq \big(-\pi^2 \Phi(t) + \alpha f(\Phi(t))\big)\, dt + \sigma dB(t).$$

Define $z(t)$ to be the one-dimensional process that verifies

$$dz = \left(-\pi^2 z + \alpha f(z)\right) dt + \sigma dB, z(0) = z_0$$

Define $z(t)$ to be the one-dimensional process that verifies

$$dz = (-\pi^2 z + \alpha f(z))\, dt + \sigma dB, z(0) = z_0$$

Comparison principle $\rightarrow \Phi(t) \geq z(t)$ as long as Φ is defined.

Define $z(t)$ to be the one-dimensional process that verifies

$$dz = (-\pi^2 z + \alpha f(z))\, dt + \sigma dB, z(0) = z_0$$

Comparison principle $\rightarrow$ $\Phi(t) \geq z(t)$ as long as Φ is defined.
We have to prove
Lema. Let z be the solution of

$$dz = (-\pi^2 z + f(z))\, dt + \sigma dB \tag{1}$$

$$z(0) = 0.$$

Then z explodes in finite time with probability one.
Proof.
Apply the *Feller Test for explosions*.

An idea of the reason of the explosion

Let $y(t) := z(t) - \sigma B(t)$

An idea of the reason of the explosion

Let $y(t) := z(t) - \sigma B(t)$

y verifies a random differential equation (i.e. an Ordinary differential equation for each ω), in fact

$$dy = -\pi^2(y(t) + \sigma B(t)) + f(y(t) + \sigma B(t))\, dt$$

Numerical Approximations

Gyöngy, 1998, 1999 (Globally Lipschitz case) Allen, Novosel and Zhang, 1998 (Linear case)

Problems:

- Lack of regularity
- Lack of boundedness
- The solution is defined just locally

Numerical Approximations

Gyöngy, 1998, 1999 (Globally Lipschitz case) Allen, Novosel and Zhang, 1998 (Linear case)
Problems:

- Lack of regularity
- Lack of boundedness
- The solution is defined just locally

Known methods for SPDEs are based in discretizations in time and space with a uniform mesh

Numerical Approximations

Gyöngy, 1998, 1999 (Globally Lipschitz case) Allen, Novosel and Zhang, 1998 (Linear case)

Problems:

- Lack of regularity
- Lack of boundedness
- The solution is defined just locally

Known methods for SPDEs are based in discretizations in time and space with a uniform mesh

Not suitable in this case since we have a singularity at a random time.

Numerical Approximations

Gyöngy, 1998, 1999 (Globally Lipschitz case) Allen, Novosel and Zhang, 1998 (Linear case)
Problems:

- Lack of regularity
- Lack of boundedness
- The solution is defined just locally

Known methods for SPDEs are based in discretizations in time and space with a uniform mesh

Not suitable in this case since we have a singularity at a random time.

Alternative: Semidiscretization in space.

Numerical Approximations

We discretize the space variable with second order finite differences for the drift and integration in space for the noise, arriving to the following system of SDEs

$$du_i = \frac{1}{h^2}(u_{i+1} - 2u_i + u_{i-1})dt + f(u_i)dt + \frac{\sigma}{\sqrt{h}}dw_i, \quad 2 \leq i \leq n-1.$$

$$u_1 = u_n = 0,$$

$$u_i(0) = u_0(ih), \quad 1 \leq i \leq n.$$

Equivalently

$$dU = (-AU + f(U))\, dt + \frac{\sigma}{\sqrt{h}}\, dW, \quad U(0) = U^0. \qquad (*)$$

Properties of the semidiscrete scheme

Theorem

Let f be a nonnegative, convex function such that

$$\int^{\infty} \frac{1}{f} < \infty.$$

Then, for every nonnegative initial datum $U^0 \geq 0$ the solution U to (*) blows-up in finite (random) time T^n with

$$\mathbb{P}^{U^0}(T^n < \infty) = 1.$$

Convergence of the Numerical Scheme

Theorem

Assume f is a nonnegative convex function with $\int \frac{1}{f} < \infty$. Let u be the solution to (P) and u^n its (semidiscrete) numerical approximation given by (*). Then

1. *For every $p \geq 1$ and for every $T > 0$ there exists a constant $K = K(p, T)$ such that*

$$\sup_{0 \leq t \leq T} \sup_{x \in [0,1]} \mathbb{E}(|u^n(t,x) - u(t,x)|^{2p} \mathbf{1}_{\{t \leq R_M \wedge R_M^n\}}) \leq \frac{K}{n^p}.$$

2. *For every $M \geq 0$ $\|u^n - u\|_{L^\infty([0,T \wedge R_M] \times [0,1])}$ converges to zero almost surely as $n \to \infty$.*

Numerical Approximations

Next we need to integrate the system of SDE

Numerical Approximations

Next we need to integrate the system of SDE

Adaptivity is required!

Numerical Approximations

Next we need to integrate the system of SDE

Adaptivity is required!

Fixed time step gives rise to solutions defined for every time. Not desirable!!

Numerical Approximations

Next we need to integrate the system of SDE

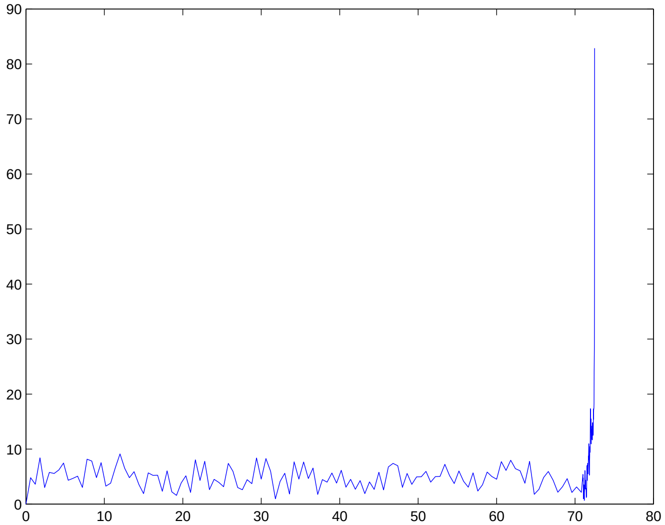
Adaptivity is required!

Fixed time step gives rise to solutions defined for every time. Not desirable!!
We adapt the time step with the following strategy

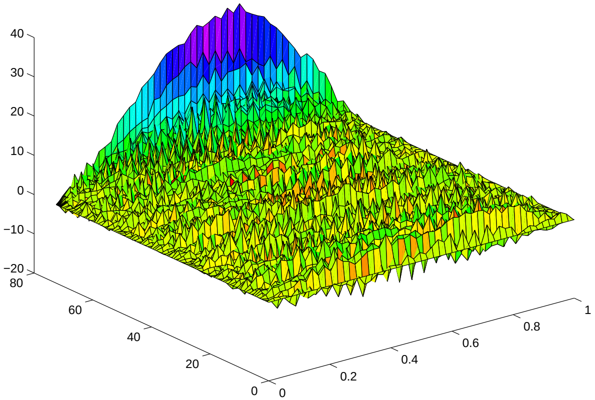
$$\Delta t_j = t_{j+1} - t_j = \frac{\lambda}{f(\sum u_i)}$$

Same questions as before....

The evolution of the L^∞ norm



A complete picture of the solution



Future work

In wich sence the perturbed solution is close to the deterministic one?

We want to prove the following

- Assume u_0 is such that the solution of the equation with $\sigma = 0$ and initial data u_0 blows-up in finite time $T(u_0)$, then

$$T_\sigma(u_0) \to T(u_0), \quad (\sigma \to 0).$$

- Assume u_0 is such that the solution is global, then

$$T_\sigma(u_0) \to \infty, \quad \text{exponentially fast}$$

$$\frac{T_\sigma(u_0)}{\mathbb{E}(T_\sigma(u_0))} \to Z.$$

$Z \sim \mathcal{E}(1)$.

Future work

In wich sence the perturbed solution is close to the deterministic one?

We want to prove the following

- Assume u_0 is such that the solution of the equation with $\sigma = 0$ and initial data u_0 blows-up in finite time $T(u_0)$, then

$$T_\sigma(u_0) \to T(u_0), \quad (\sigma \to 0).$$

- Assume u_0 is such that the solution is global, then

$$T_\sigma(u_0) \to \infty, \quad \text{exponentially fast}$$

$$\frac{T_\sigma(u_0)}{\mathbb{E}(T_\sigma(u_0))} \to Z.$$

$Z \sim \mathcal{E}(1)$.

THANKS!