An anisotropic GLS-stabilization method for the finite element solution of the Stokes problem

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Abstract

The usual methods of stabilization of the pressure in the numerical solution of incompressible flow problems through the finite element method ([1], [3], [4], [6], [7]) lose their efficiency when used together with highly anisotropic meshes, which are frequently employed in applications of the finite element method. Specific methods have been derived for these situations, in which the elemental (scalar) stabilization parameters are designed taking into account the anisotropy of the mesh ([5], [8]).

In this work, an anisotropic method for the stabilization of the pressure in incompressible flow problems is presented, which is based on the usual Galerkin-Least-Squares method ([7]). The stabilization is not achieved in this case from a unique scalar parameter in each element but from an elemental matrix of stabilization parameters, which is given by the Jacobian matrix of the isoparametric transformation from the reference element to the current one. This way, not only the anisotropy of the mesh is taken into account but also the spatial directions with respect to which this anisotropy holds. Stability and convergence of this method are proved in an norm which is adequate for the anisotropy of the mesh.

In the case of Cartesian grids of rectangular elements, a stabilized formulation is obtained with different stabilization parameters in each spatial direction within each element; these parameters are computed in terms of the element size in that direction (not of the overall element size). This scheme is similar to that introduced in [2].

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