Multiscale enrichment and stabilization of the Stokes problem.

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ABSTRACT

In this work we present a new stabilized finite element method for the Stokes problem. The method is obtained by enriching the velocity space with multiscale functions (i.e., local but not bubble-like functions, see [3]), which are solutions of a differential problem with the residual of the momentum equation as right hand side in each element, completed with special boundary conditions which are solutions of an elliptic ODE (governed by the restriction of the differential operator to the edge) on the element boundary. The impact produced by these enrichment functions in the Galerkin formulation is evaluated by performing static condensation, which leads to a method containing all the terms of a GLS method (cf. [4]), plus non-standard jump terms in the internal edges of the triangulation, each one with its own stabilization parameter.

One of the main particularities of the proposed methods is that the stabilization parameter associated to the jump terms on the internal edges is known exactly, since it comes from the solution of an elliptic ODE in the boundary, which is not the case for other methods containing jump terms on the internal edges (cf. [1,4]).

Besides the general framework, we present specific stabilized finite element methods using equal order P^1/P^1 polynomials for velocity and pressure and the *simplest* element, namely, continuous P^1 polynomials for the velocity and piecewise constant elements for the pressure. The latter has been possible to stabilize only due to the fact that we are using enrichment functions which do not vanish on the element boundaries (indeed, in [2] is remarked that the P^1/P^0 element can not be stabilized by adding bubble functions to the velocity space). For both choices of elements we give optimal order convergence results and several numerical experiments confirming these convergence results.

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