Finite element approximation of convection diffusion problems using graded meshes

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ABSTRACT. We analyze the approximation of the solution of a model convection-diffusion equation. We prove that, using appropriate graded meshes, the solution is well approximated by the standard piecewise bilinear finite element method in the ε-weighted H^1-norm $\|\,.\,\|_\varepsilon$ defined as

$$\|v\|_\varepsilon^2 = \|v\|_{L^2(\Omega)}^2 + \varepsilon\|\nabla v\|_{L^2(\Omega)}^2.$$

Precisely, we consider the problem

$$\begin{aligned} -\varepsilon\Delta u + b\cdot\nabla u + cu &= f \qquad \text{in } \Omega \\ u &= 0 \qquad \text{on } \partial\Omega \end{aligned} \tag{0.1}$$

where $\Omega = (0,1)^2$ and $\varepsilon > 0$ is a small parameter. We prove that, on appropriate graded meshes,

$$\|u - u_h\|_\varepsilon \leq C\log(1/\varepsilon)\frac{(\log(1/\varepsilon) + \log N)}{\sqrt{N}}.$$

where u_h is the standard piecewise bilinear approximation of u on a graded mesh $\mathcal{T}_h$ (where $h > 0$ is a parameter arising in the definition of the mesh), N denotes the number of nodes, C is a constant independent of ε and N.

Observe that this error estimate is almost optimal, i. e., up to logarithmic factors, the order with respect to the number of nodes is the same as that obtained for a smooth function on uniform meshes.

Consequently, the graded meshes seems an interesting alternative to the well known Shishkin's meshes which provides also almost optimal order. Indeed, from some numerical experiments the graded meshes procedure seems to be more robust in the sense that the numerical results are not strongly affected by variations of parameters defining the meshes.

Finally, we present some numerical examples showing the good behavior of our method.