

ON THE COMPUTATIONAL COMPLEXITY OF THE HILBERT POLYNOMIAL

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We study the computational complexity of the problem of computing the Hilbert polynomial of a complex projective variety V , given by a list of homogeneous input polynomials. In case V is smooth and equidimensional, we show that the problem can be reduced in deterministic polynomial time (in the sparse size of the input polynomials) to the problem of counting the number of complex common zeros of a finite set of multivariate polynomials. The reduction algorithm is based on ideas from complexity theory (elimination of generic quantifiers) and on a formula expressing the coefficients of the Hilbert polynomial in terms of the degrees of certain degeneracy loci. This formula is derived from well-known results in Schubert calculus and the Hirzebruch-Riemann-Roch theorem. The problem of counting the number of solutions of a system of polynomial equations can be solved in polynomial space. The general problem of computing the Hilbert polynomial of a homogeneous ideal can be shown to be at least as hard as the homogeneous ideal membership problem, a problem known to be polynomial-space hard.

Reference:

P. Bürgisser and M. Lotz. *The complexity of computing the Hilbert polynomial of smooth equidimensional complex projective varieties*. www.arxiv.org/abs/cs/cs.CC/0502044.