QUADRATIC NEWTON ITERATION FOR SYSTEMS WITH MULTIPLE ZEROES AND CLUSTERS OF ZEROES

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Newton's iterator is one of the most popular components of polynomial equation system solvers, either from the numeric or symbolic point of view. This iterator usually handles smooth situations only (when the Jacobian matrix associated to the system is invertible). This is often a restrictive factor. Generalizing Newton's iterator is still a challenging problem: how to design an efficient iterator with a quadratic convergence even in degenerate cases?

In this talk, we will present a symbolic algorithm for a I-adic topology when the ideal I can be chosen generic enough: compared to a smooth case we prove quadratic convergence with a small overhead that grows with the square of the multiplicity of the root.

Then we will present a numeric generalization of this algorithm to analytic maps. We will restrict to situations where the analytic map has corank one at the multiple isolated zero, which has embedding dimension one in the frame of deformation theory. These situations are the least degenerate ones and therefore most likely to be of practical significance. More generally, we define clusters of embedding dimension one. We provide a criterion for locating such clusters of zeroes and a fast algorithm for approximating them, with quadratic convergence. In case of a cluster with positive diameter our algorithm stops at a distance of the cluster which is about its diameter. These results are in the vein of the α-theory for simple zeroes, that was initiated by M. Shub and S. Smale in the beginning of the eighties.

The numeric generalization is joint work with Marc Giusti, Bruno Salvy and Jean-Claude Yakoubsohn.