INSOLVABILITY OF EQUATIONS IN FINITE TERMS

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Abel, Galois, Liouville, Picard, Vessiot, Kolchin and others found a lot of results about solvability and insolvability of equations in finite terms. According to them, algebraic equations are usually not solvable by means of radicals. Ordinary linear differential equations and holonomic systems of linear differential equations in partial derivatives are not usually solvable by quadratures. Galois theory belongs to algebra. In fact results about insolvability of differential equations belong to differential algebra (and are also purely algebraic). About 30 years ago I constructed a topological version of Galois theory for functions in one complex variable. According to it, there are topological restrictions on the way the Riemann surface of a function representable by quadratures covers the complex plane. If the function does not satisfy these restrictions, then it is not representable by quadratures. Beside its geometric clarity the topological results on nonrepresentability of functions by quadratures are stronger than the algebraic results. Recently I have constructed a multi-dimensional topological version of Galois theory.