A SHORT PROOF OF LYNDON AND NEWMAN'S RESULT THAT $c = [x, y]$ IS NOT A PRODUCT OF TWO SQUARES IN THE FREE GROUP F WITH BASIS $\{x, y\}$ USING THE LYNDON IDENTITY THEOREM

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ABSTRACT. Unlike [1-4,6-9], we use [5].

If $c = a^2b^2 = (ab)b^{-1}(ab)b$, $ab \in \mathrm{F}'$ so $ab = \prod_{i=1}^{n} d_i c^{\epsilon_i} d_i^{-1}$ ($d_i \in \mathrm{F}$, $\epsilon_i = \pm 1$) and $1 = c^{-1} \prod_{i=1}^{n} d_i c^{\epsilon_i} d_i^{-1} \prod_{i=1}^{n} b^{-1} d_i c^{\epsilon_i} d_i^{-1} b$ has $2n+1$ factors.

REFERENCES

[1] J.A. Barmak. *The winding invariant*. arXiv:1904.10072.

[2] M. Culler. *Using surfaces to solve equations in free groups*. Topology 20(1981), 133-145.

[3] C.C. Edmunds. *On the endomorphism problem for free groups*. Comm. in Algebra 3(1975), 1-20.

[4] R.Z. Goldstein, E.C. Turner. *Applications of topological graph theory to group theory*. Math. Zeitschrift 165(1979), 1-10.

[5] R.C. Lyndon. *Cohomology theory of groups with a single defining relation*. Ann. Math. 52(1950), 650-665.

[6] R.C. Lyndon, M.F. Newman. *Commutators as products of squares*. Proc. Amer. Math. Soc. 39(1973), 267-272.

[7] R.C. Lyndon, P.E. Schupp. *Combinatorial group theory*. Springer (1977).

[8] S. Sarkar. *Commutators and squares in free groups*. Alg. Geom. Top. 4(2004), 595-602.

[9] M.J. Wicks. *The equation $x^2y^2 = g$ over free products*. In Proc. of the Second Congress of the Singapore National Academy of Science (1973), 238-248.

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