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The rectifiable distance in the unitary Fredholm group

Let $U_c(\mathcal{H}) = \{u : u \text{ unitary and } u - 1 \text{ compact}\}$ stand for the unitary Fredholm group. We prove the following convexity result. Denote by d_∞ the rectifiable distance induced by the Finsler metric given by the operator norm in $U_c(\mathcal{H})$. If $u_0, u_1, u \in U_c(\mathcal{H})$ and the geodesic β joining u_0 and u_1 in $U_c(\mathcal{H})$ verify $d_\infty(u, \beta) < \pi/2$, then the map $f(s) = d_\infty(u, \beta(s))$ is convex for $s \in [0, 1]$. In particular the convexity radius of the geodesic balls in $U_c(\mathcal{H})$ is $\pi/4$. The same convexity property holds in the p-Schatten unitary groups $U_p(\mathcal{H}) = \{u : u \text{ unitary and } u - 1 \text{ in the } p\text{-Schatten class}\}$, for p an even integer, $p \geq 4$ (in this case, the distance is strictly convex). The same results hold in the unitary group of a C^*-algebra with a faithful finite trace. We apply this convexity result to establish the existence of curves of minimal length with given initial conditions, in the unitary orbit of an operator, under the action of the Fredholm group. We characterize self-adjoint operators A such that this orbit is a submanifold (of the affine space $A + \mathcal{K}(\mathcal{H})$, where $\mathcal{K}(\mathcal{H})$=compact operators).