# TIME-FREQUENCY UNCERTAINTY PRINCIPLES FOR SHIFT INVARIANT SPACES

ABSTRACT. The Balian-Law Theorem states that if a Gabor system generated by a function $g$ is Riesz basis for $L^2(\mathbb{R})$, then $g$ cannot be well localized in both time and frequency. Specifically, $\|xg(x)\|_2\|\omega\hat{g}(\omega)\|_2 = \infty$. For shift invariant spaces, time-frequency obstructions also occur. For example, we will show that if $\phi$ generates a principal shift invariant space which is also $\frac{1}{n}$ invariant for some $n > 1$, then $\phi$ cannot be well localized in both time and frequency. For example, $\|xg(x)\|_2 = \infty$. We will present these time frequency obstructions for $\phi$ and show that the results are optimal.