① Let $R$ be a ring, $G$ be a group. For $r \in R$ with $r^N = 0$ and $f \in R[G]$ with $rf = fr$

(1) $1 - rf$ is a unit

On monday we only considered unit of the form

(2) $1 - rg$ with $g \in G$.

Show that any unit of the form (1) is a product of units of the form (2).

② Let $C_5$ be the cyclic group of order 5. Let $t$ be a generator of $C_5$.

Show that $1-t-t^4$ is a unit in $\mathbb{Z}[C_5]$.

Show also that in $K_1(\mathbb{Z}[C_5])$ we have $[1-t-t^4] \neq [\pm g]$ for all $g \in C_5$, i.e., that $1-t-t^4$ is even in K-theory different from the canonical units.

<u>Hint:</u> Use $\mathbb{Z}[C_5] \xrightarrow{t \mapsto e^{2\pi i/5}} \mathbb{C}$ and $\det : K_1(\mathbb{C}) \longrightarrow \mathbb{C}$.

③ Let $P$ be a finitely generated free $R$-module. Find a bounded below chain complex of finitely generated free $R$-modules $C$, s.t.

a) $P \simeq C$ b) $C$ is dominated by a finite length chain complex of finitely generated free $R$-modules.

④ Let $H$ be a finite group. Let $P := \mathbb{Q}$ with the trivial $H$-action.

Show that $P$ is projective as a $\mathbb{Q}[H]$-module.

Show that $P$ does not have a finite length resolution by finitely generated free $R$-Modules.

<u>Hint:</u> Show first $[P] \notin \underset{\cong\ \mathbb{Z}}{K_0(\mathbb{Q})} \subseteq K_0(\mathbb{Q}[H])$

⑤ Let

$$\begin{aligned} \{\pm 1\} \oplus G_{ab} &\xrightarrow{A_1} K_1\,\mathbb{Z}[G] \\ (\varepsilon, g) &\longmapsto [\varepsilon \cdot g] \end{aligned}$$

be defined by

Show that $A_1$ is (split) injective.