WATACBA - Buenos Aires, November 2011

ABSTRACTS

Michal Adamaszek (University of Warwick)

Natural splittings of independence complexes and applications

I will discuss a method of decomposing independence complexes of graphs into wedge sums based on simple edge and vertex manipulations in the underlying graph. As a main application we determine the independence complexes of powers of cycles, which answers an open problem of D. Kozlov. Time permitting I will mention other applications of the method, for instance to the hard square models on some small grids.

Andrés Angel (Universidad de los Andes)

String topology of orbifolds

String topology is the study of the algebraic structures that govern the interactions of loops in a manifold. The relation with Hochschild homology and cohomology has been studied from different aspects and in this talk I will explain how to generalize this connections to non-simply connected spaces. The motivation of this work is to extend the known construction of string topology to orbifolds.

Eric Babson (University of California Davis)

A central limit theorem for certain patterns in permutations

The distribution of lengths of longest increasing sequences in permutations has been extensively studied. The length is typically around the square root of the total length and the distribution has been characterized rather explicitly. More recently the easier case of alternating sequences was considered. The length here is typically a fraction of the total length and the distribution follows a central limit theorem. I will discuss other patterns behaving like the alternating one.

This is based on joint work with Abrams, Landau, Landau and Pommersheim.

Natalia Viana Bedoya (Universidade Federal de Sao Carlos)

Decomposability problem of branched coverings

Given a branched covering of degree d between closed surfaces, it determines a collection of partitions of d, the branch datum. In this work we show that any branch data is realized by an indecomposable primitive branched covering on a connected closed surface N with $\chi(N) \leq 0$. This shows that decomposable and indecomposable realizations may coexist. Moreover, we characterize the branch data of a decomposable primitive branched covering. It is a joint work with Daciberg Lima Goncalves.

Bruno Benedetti (Royal Institute of Technology (KTH))

Metric geometry and collapsibility

A simplicial complex can be turned into a (piece-wise Euclidean) metric space by assigning the same length to all edges. We show that if a complex becomes a CAT(0) metric space with this metric, then it is combinatorially collapsible. This is a discrete analog of the classical theorem that CAT(0) complexes are contractible. As an application, we construct some manifolds different than balls that admit a collapsible triangulation. This contrasts a 1939 result of Whitehead. Also, it shows that discrete Morse theory can be sharper than smooth Morse theory in bounding the homology of a manifold.

If time permits, we discuss extensions to different metrics and applications to Hirsch diameter bounds.

This is joint work with Karim Adiprasito (arxiv.org/abs/1107.5789).

Anders Björner (Royal Institute of Technology (KTH))

Topological combinatorics – glimpses and comments

As an introduction to this area I will describe a few cases where topology has been an essential tool for solving problems in discrete mathematics. Also, some current trends will be discussed.

José Calcines (Universidad de La Laguna)

Topological complexity and the homotopy cofibre of the diagonal map

The topological complexity of a space X, $TC(X)$, is the sectional category (or Schwarz genus) of the end-points evaluation fibration $\pi_X : X^I \to X \times X$, $\pi_X(\alpha) = (\alpha(0), \alpha(1))$. This homotopical invariant was defined by M. Farber, giving a topological approach to the robot motion planning problem. In robotics if one regards the topological space X as the configuration space of a mechanical system, the motion planning problem consists of constructing a program or a devise, which takes pairs of configurations $(A, B) \in X \times X$ as an input and produces as an output a continuous path in X, which starts at A and ends at B. Broadly speaking, $TC(X)$ measures the discontinuity of any motion planner in the space. Further developments of the topological complexity have proved to be a very interesting homotopical invariant. It not only interacts with robotics but also with deep problems arising in algebraic topology. The goal of this talk is to give some partial results on the comparison of the two invariants $TC(X)$ and $cat(C_{\Delta_X})$, where C_{Δ_X} denotes the homotopy cofibre of the diagonal map $\Delta_X : X \to X \times X$. In particular we obtain as a consequence that the immersion problem of $\mathbb{R}P^n$ is equivalent to the computation of $cat(C_{\Delta_{\mathbb{R}P^n}})$.

Moira Chas (Stony Brook University)

String topology and three manifolds

In the late nineties, we found in joint work with Dennis Sullivan that for an oriented manifold M there is a Lie algebra structure on the equivariant homology of the mapping space of the circle into M, equivariant with respect to the rotation of the domain circle. When M is a surface this reduces to the Goldman bracket on the free abelian group generated by free homotopy classes of oriented closed curves on M.

Suppose now that M is an oriented three manifold with contractible universal covering space. One knows such a manifold decomposes into pieces along incompressible two dimensional tori and that these pieces are either hyperbolic or Seifert fibrations. We will discuss how that the graded Lie algebra structure of String Topology determines the combinatorial structure of the torus decomposition.

To do this, we have to extend the Goldman bracket for surfaces to two dimensional orbifolds. The key result in proving our theorem is that the String Topology bracket, as well as the Goldman bracket for orbifolds "counts" mutual intersections and self-intersections of curves and surfaces in the three manifold.

Hellen Colman (Wright College)

Lusternik-Schnirelmann category for orbifolds as groupoids

We propose a new numerical invariant for topological groupoids which generalizes the Lusternik-Schnirelmann category of topological spaces. This number is invariant under Morita equivalence, then yields a well defined LS-category for orbifolds.

An orbifold map is given by an equivalence class of generalized maps between groupoids. These generalized maps are obtained by formally inverting essential equivalences in the groupoid $\mathcal{G}$. We develop a notion of Morita homotopy between generalized maps using the Haefliger $\mathcal{G}$-paths based on partitions of the unit interval. We prove that the LS-category of a groupoid is a Morita homotopy invariant.

We present a bicategorical approach to discuss a Quillen model structure on the category of orbifolds where the weak equivalences are our Morita homotopy equivalences.

Estimates for the LS-category of an orbifold relate to other numerical invariants such as orbifold Euler characteristic and groupoid cardinality.

Péter Csorba (Rényi Institutes)

The topological reason of combinatorial formulas

I will explain the topological background of Brouwer's theorem, and in some sense similar results around the independence complex.

Rafael Diaz (IMA - Universidad Sergio Arboleda)

The Rota-Möbius function and its generalizations

We propose a new generalization of the Möbius function for locally finite posets, that applies to locally finite acyclic categories. We show that our construction is closely related to the Satake's Euler characterisic of an orbifold. We compare our definition with other possible generalizations such as the Leinster's Euler characteristic of a weighted (finite) category.

Jean-Marie Droz (University of Breman)

Homotopy theories of graphs

We describe the construction of Quillen model structures on a category of graphs. A basic example is that the core of a graph can be interpreted as its homotopy type.

Etienne Fieux (Institut Mathematique de Toulouse)

Homotopy for finite graphs in terms of foldings

A vertex x of a graph G is said *dismantlable* if there is another vertex a which contains all neighbors of x; the deletion of a dismantlable vertex is called a *folding*. This notion is closely related to the deletion of an *irreducible* point (or *beat point*) in a poset. We study the homotopy type of a graph (as introduced by A. Dochtermann) in terms of foldings and we prove in particular that two posets have the same homotopy type if, and only if, their comparability graphs have the same homotopy type (actually, we get more precise statements). The *clique poset* of a graph G is the set of all its induced complete subgraphs ordered by the inclusion; we also get in particular that two reflexive graphs have the same homotopy type if, and only if, their clique posets have the same homotopy type. These notions of homotopy are also related to the notion of *strong homotopy* introduced by J. Barmak and G. Minian for simplicial complexes. We illustrate it by studying the relation between the clique poset of the graph $hom(G, H)$ of morphisms from a graph G to a graph H with the face poset of the polyhedral Hom complex $Hom(G, H)$. This refines well known results about the behaviour of the Hom complex with foldings.

Pavol Hell (SFU - Canada)

Graph Partitions

I will describe attempts at characterizing partition types that can be described by finitely many forbidden induced subgraphs. Examples include many partitions that occur in the study of perfect graphs. This is joint work with Tomas Feder, Wing Xie, and Shekoofeh Nekooei Rizi, and will also include work by Juraj Stacho, Geordie Schell and others.

Daniel Jaume (Universidad Nacional de San Luis)

Walks on Unitary Cayley Graphs and Sums of Units in $\mathbb{Z}_n$

The group of units in the ring $\mathbb{Z}_n$ of the residue classes $\mod n$ consists of the residues $a \mod n$ with $(a, n) = 1$ (i.e. residues coprimo with n). we denote with $\mathbb{U}_n$ to the group of units of $\mathbb{Z}_n$.

In 2009, Sander (see [1]) determined the number of representations of a fixed residue class $\mod n$ as (ordered) sum of two units in $\mathbb{Z}_n$.

In this work, we prove the following result using graphs theory technics:

Theorem 1. *Given $0 < k, n \in \mathbb{Z}$ the number of k-uples in $\mathbb{U}_n^k$, $(u_1, u_2, \ldots, u_k) \in \mathbb{U}_n^k$ which are solutions of*

$$u_1 + u_2 + \cdots + u_k \equiv_n 0$$

is

$$\frac{\varphi\left(rad(n)\right)}{rad(n)} \left(\frac{n}{rad(n)}\right)^{k-1} \prod_{p|n} \left((p-1)^{k-1} - (-1)^{k-1}\right)$$

Here we uses Nathanson notation (see [2]): the radical of n is the product of the primes which divides n:

$$rad(n) = \prod_{p|n} p$$

This is a generalization of the result of Sander (case $k = 2$).

References

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Jakob Jonsson (Royal Institute of Technology (KTH))

Hom complexes and test graphs revisited

In a seminal paper from 1978, László Lovász used topological methods to prove Kneser's conjecture, a statement about the chromatic number of certain graphs. In his proof, Lovász associated to each graph H a certain simplicial complex and showed that the connectivity degree of this complex provides a lower bound on the chromatic number $\chi(H)$ of H. As it turned out, this complex is homotopy equivalent to a certain cell complex whose vertices are homomorphisms from the complete graph K_2 to H. Replacing K_2 with an arbitrary graph G, one obtains the so-called Hom complex, denoted $\mathrm{Hom}(G, H)$, associated to the pair (G, H). A natural question to ask is to what extent topological properties of $\mathrm{Hom}(G, H)$ can tell us anything useful about $\chi(H)$. The concept of a test graph provides a partial answer to this question. Roughly speaking, a graph G is a test graph if, for every graph H, the difference $\chi(H) - \chi(G)$ is bounded from below (up to a shift) by the connectivity degree of $\mathrm{Hom}(G, H)$. Hoory and Linial constructed an example showing that not all graphs are test graphs. Yet, by the work of Babson, Kozlov, Dochtermann, Schultz and others, there are several known classes of test graphs, e.g., complete graphs and odd cycles. One of the main goals of the talk is to question whether the concept of a test graph really is the one that we are looking for. One may imagine a graph G with the ostensibly weaker property that $\chi(H) - \chi(G)$ is bounded from below only by a small constant, say $1/k$, times the connectivity degree of $\mathrm{Hom}(G, H)$. Such a graph G would not be a test graph in the usual sense, but it might well be the case that G provides us with better bounds on $\chi(H)$ than conventional test graphs. In fact, if we step outside the realm of graphs and allow for more general hypergraphs, it turns out that we can construct hypergraphs G with this very property. This is in alignment with well-known analogous applications of obstruction theory to Tverberg-like theorems.

Martina Kubitzke (Universität Wien)

Poset fiber theorems and some applications

We study topological properties of the lattices of non-crossing partitions of types A and B and the poset of injective words. In particular, it is proved that those posets are doubly homotopy Cohen-Macaulay. This extends the well-known results that those posets are homotopy Cohen-Macaulay. Our results rely on a new poset ber theorem for doubly homotopy Cohen-Macaulay posets. Similar to the classical poset fiber theorem by Quillen for homotopy Cohen-Macaulay posets, this turns out to be a new useful tool to show doubly homotopyCohen-Macaulayness of a poset. We provide two more applications to certain complexes of injective words which were originally introduced by Jonsson and Welker. This is joint work with Myrto Kallipoliti.

Frank Lutz (Technische Universität Berlin)

Random methods in discrete topology and the complicatedness of triangulations

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Criel Merino (Universidad Nacional Autónoma de México)

On the structure of the H-vector of paving matroids

A paving matroid is one in which all it's circuits have size at least the rank of the matroid. The interest about paving matroids goes back to 1976 when Dominic Welsh ask if most matroids are paving. In this talk we will show some properties for the class of paving matroids. Our main result is that the h-vector of a paving matroid is a pure O-sequence. Where the h-vector of a matroid is an easy transformation of the face enumerator of the simplicial complex coming from the independent sets in the matroid. While multicomplex is a natural generalization of simplial complex to multisets. The multicomplex is pure if all the facets have the same sizes. A sequence of numbers is an (pure) O-sequence if it's the face enumerator of a (pure) multicomplex.

Jasper Moller (University of Copenhagen)

Relaxed vertex colorings of simplicial complexes

An r-coloring of a graph paints the vertices of the graph from a palette of r colors avoiding monochrome 1-simplices. An (r, s)-coloring of a simplicial complex paints the vertices of the complex from a palette of r colors avoiding monochrome s-simplices where s can be 1 or more. The combinatorial problem of finding an (r, s)-coloring of a given simplicial complex is equivalent to a topological lifting problem. This concept of relaxed colorings lead to some higher dimensional open problems related to the 4-color theorem. A preprint is available here http://arxiv.org/abs/1007.0710.

Oscar Eduardo Ocampo Uribe (Institute of Mathematics and Statistics - São Paulo University)

Homotopy groups of Δ-groups formed by surface pure braids

Let $P_{n+1}(M)$ denote the pure braid group with $n + 1$ strings on a surface M, possibly M non-orientable or with boundary. Let $\mathcal{P}(M) = \{P_{n+1}(M)\}_{n\geq 0}$ be the Δ-group formed by surface pure braid groups, with faces maps induced from the Fadell-Neuwirth fibration. In this work we describe the homotopy groups $\pi_n(\mathcal{P}(M))$, for any surface M, and $n \geq 0$. As a consequence, we exhibit some subgroups of braid groups that project onto $\pi_n(\mathbb{S}2)$, for $n \geq 3$.

Hugo Rodríguez Ordóñez (Universidad Autónoma de Aguascalientes)

Dimensional Restrictions upon counterexamples to Ganea's conjecture

The long standing conjecture by Ganea on the Lusternik-Schnirelmann category was disproved in the late 1990s by means of a family of counterexamples whose least dimensional element has dimension 10. In a previous work, the authors proved that there is a 7-dimensional counterexample. In this work, we present a proof that there is no counterexample to this conjecture with dimension 6 or less. This is joint work with Don Stanley.

Martin Tancer (Charles University)

Embeddability obstructions to intersection patterns of convex

The Helly theorem states that if there are n convex sets in d-dimensional space with n at least $d + 1$ such that the intersection of any $d + 1$ of these sets is nonempty, then the intersection of all sets is nonempty. This theorem can be seen as an evidence that the intersection patterns of convex sets cannot be arbitrary.

In the talk we discuss how obstructions to embeddability of simplicial complexes into Euclidean space influent possible intersection patterns. We also discuss some related results.

Evgeny Troitsky (Moscow State Lomonosov University)

Counting Reidemeister classes

The subject of this talk is a description of the modern state of several problems related to Reidemeister zeta function. Let $\phi : G \to G$ be an automorphism of a discrete finitely generated group G. The Reidemeister number $R(\phi)$ is the number of ϕ-conjugacy classes $g \sim xg\phi(x^{-1})$. For topological applications G is the fundamental group of a finite CW-complex.

Two principal (interrelated) questions of the field are:

(1) To identify in a natural way $R(\phi)$ and the number of fixed points of the corresponding homeomorphism $\widehat{\phi}$ of an appropriate dual space of G (twisted Burnside-Frobenius theorem, TBFT);
(2) To find classes of groups for which any automorphism has infinite $R(\phi)$ (property R-infinity).

Technics is analytical and combinatorial.

José Antonio Vilches Alarcón (Universidad de Sevilla)

Optimal and perfect discrete Morse functions on 3*-manifolds*

This work is focused on characterizing the existence of a perfect/optimal discrete Morse function on a triangulated 3-manifold M. It is established in terms of the existence of such kind of function on a spine K of M, that is, a 2-subcomplex K such that $M - \Delta$ collapses to K, where Δ is a tetrahedron of M. Also, attending to the decomposition of every 3-manifold into prime factors, we prove that if every prime factor of M admits a perfect discrete Morse function, then M admits such kind of function.

Antonio Viruel (University of Málaga)

Lifting group actions in graphs to groups actions in homotopy

The main problem we address in this lecture is

Problem 1. ***Realisation problem of self homotopy equivalences:*** [6, 1] *Given a finite group G, find a (rational) space X such that the group of self homotopy equivalences of X, $\mathcal{E}(X)$, is isomorphic to G.*

So far, the only known general approach to Problem 2 was when $G \cong \mathrm{Aut}(\pi)$ and then $X = K(\pi, 1)$. We prove:

Theorem 2. *Let G be a finite group. Then there exist infinitely many homotopy types X such that $\mathcal{E}(X) \cong G$.*

In order to prove that result, we introduce a new technique which we hope can be useful in may other areas. We construct a functor from a subcategory of graphs to topological spaces that preserves group actions. Let $\mathcal{G}raph_{full}$ be the category whose objects are connected (simple) graphs, and morphisms are injective full homomorphisms. We define $\mathcal{M}\colon \mathcal{G}raph_{full} \rightsquigarrow DGA$ (Differential Graded Algebras), and therefore to the homotopy category, such that given a graph $\mathcal{G}$, $\mathrm{Aut}(\mathcal{G}) \cong \mathcal{E}(\mathcal{M}(\mathcal{G}))$.

We will also briefly discuss two applications of our technique.

Problem 2. ***Existence of nontrivial functorial semi-norms on singular homology:*** [4] *Let $d \in \mathbb{N}$. Does a functorial semi-norm on singular homology in degree d take only the values* 0 *and ∞ on homology classes of simply connected spaces?*

Problem 3. ***Isomorphism problem via Representation Theory:*** *Let G be a finite group and $\mathcal{C}$ be a category. Is the isomorphism type of G determined by the set of $\mathcal{C}$-objects that admits an effective G-action?*

Recent work by Crowley-Loeh [2] settles Problem 4 in negative by constructing functorial semi-norms of degree $d \equiv 0 \mod 4$. This problem is related with the existence of "inflexible manifolds" (manifolds in which every self map has degree -1, 0 or 1). We prove:

Theorem 3. *There exist infinitely many functorial semi-norms on singular homology in degree $d \equiv 2 \mod 4$ that are positive and finite on certain homology classes of simply connected spaces.*

Finally, the work of Hertweck [5] shows that Problem 5 has a negative answer when considering effective linear actions. By requiring a more restrictive set of effective actions, we settle Problem 5 in affirmative. We prove:

Theorem 4. *Let G and H be finite groups, and (A, d) be a finitely generated rational DGA. Then the following statements are equivalent:*

- *G and H are isomorphic.*
- *$G \leq \mathrm{Aut}(A, d)$ if and only if $H \leq \mathrm{Aut}(A, d)$.*

References

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Russ Woodroofe (Washington University in St. Louis)

Antichain cutsets of strongly connected posets

An antichain cutset for a poset is an antichain which intersects every maximal chain. Rival and Zaguia showed that the antichain cutsets of a finite Boolean lattice consist exactly of the level sets. We show that a similar characterization holds for the antichain cutsets of any discrete and strongly connected poset. Thus, for example, any Cohen-Macaulay poset has antichain cutsets consisting of its level sets. This is joint work with Stephan Foldes.

Inna Zakharevich (Massachusetts Institute of Technology)

Scissors Congruence as K-theory

The traditional question of scissors congruence is purely geometric: given two polytopes, when is it possible to dissect them into pairwise congruent pieces? The classical work of Dupont and Sah translated this geometry into questions of group homology, and used group homological tools in their computations. However, in the course of their computations, several tantalizing connections to algebraic K-theory emerge: for example, scissors congruence groups of hyperbolic 3-space can be placed into a long exact sequence with the groups $K_2(\mathbf{C})^-$ and $K_3(\mathbf{C})^-$. In order to try and make sense of these computations we go back to the foundations of the scissors congruence problem in hopes of rephrasing it in terms of algebraic K-theory. To do this we draw our attention away from the geometric data and focus on more combinatorial notions of ”covering” and ”intersection” in order to consruct a Waldhausen category whose K-theory spectrum will, on the 0 level, be exactly the scissors congruence groups of Dupont and Sah.