On the 1-type of Waldhausen K-theory

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Goal

- Understanding K_1 in the same clear way we understand K_0.

We use the following notation for the basic structure of a Waldhausen category **W**:

- Zero object $*$.
- Weak equivalences $A \xrightarrow{\sim} A'$.
- Cofiber sequences $A \rightarrowtail B \twoheadrightarrow B/A$.

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Waldhausen categories

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K_0 of a Waldhausen category

The abelian group $K_0\mathbf{W}$ is generated by the symbols

- $[A]$ for any object A in $\mathbf{W}$.

These symbols satisfy the following relations:

- $[*] = 0$,
- $[A] = [A']$ for any weak equivalence $A \xrightarrow{\sim} A'$,
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The K-theory of a Waldhausen category $\mathbf{W}$ is a spectrum $K\mathbf{W}$.

The spectrum $K\mathbf{W}$ was defined by Waldhausen by using the $S.$-construction which associates a simplicial category $wS.\mathbf{W}$ to any Waldhausen category.

A simplicial category is regarded as a bisimplicial set by taking levelwise the nerve of a category.

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The nature of our algebraic model

A stable quadratic module C consists of a diagram of groups

$$C_0^{ab} \otimes C_0^{ab} \xrightarrow{\langle -,- \rangle} C_1 \xrightarrow{\partial} C_0$$

such that

- $\langle a, b \rangle = -\langle b, a \rangle$,
- $\partial\langle a, b \rangle = -b - a + b + a$,
- $\langle \partial c, \partial d \rangle = -d - c + d + c$.

The homotopy groups of C are

- $\pi_0 C = \operatorname{Coker} \partial$,
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A symmetric monoidal category

A stable quadratic module C gives rise to a symmetric monoidal category $\mathbf{smc}C$ with

- object set C_0,
- morphisms $(c_0, c_1)\colon c_0 \to c_0 + \partial c_1$ for $c_0 \in C_0$ and $c_1 \in C_1$.

The symmetry isomorphism is defined by the bracket

$$(c_0 + c_0', \langle c_0, c_0' \rangle)\colon c_0 + c_0' \xrightarrow{\cong} c_0' + c_0.$$

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The classifying spectrum

Segal's construction associates a classifying spectrum $B\mathbf{M}$ to any symmetric monoidal category $\mathbf{M}$.

The spectrum $B\mathbf{smc}C$ has homotopy groups concentrated in dimensions 0 and 1.

Moreover,

$$\begin{aligned} \pi_0 B\mathbf{smc}C &\cong \pi_0 C, \\ \pi_1 B\mathbf{smc}C &\cong \pi_1 C. \end{aligned}$$

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We define a stable quadratic module $\mathcal{D}_*\mathbf{W}$ by generators and relations which models the 1-type of $K\mathbf{W}$:

Theorem

There is a natural morphism in the stable homotopy category

$$K\mathbf{W} \longrightarrow B\mathbf{smc}\mathcal{D}_*\mathbf{W}$$

which induces isomorphisms in π_0 and π_1.

Corollary

There are natural isomorphisms

$$K_0\mathbf{W} \cong \pi_0\mathcal{D}_*\mathbf{W},$$
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The generating symbols satisfy six kinds of relations.

- The trivial relations formulas bisimplices.
- The boundary relations formulas bisimplices.
- Composition of weak equivalences formula bisimplex.
- Weak equivalences of cofiber sequences formula bisimplex.
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Related work

- Nenashev's presentation of K_1 of an exact category.
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New features of our approach

- Valid for Waldhausen categories.
- Use of strict algebraic structures of optimal nilpotency degree.
- Generators and relations are given by objects, weak equivalences, and cofiber sequences.
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- The multiplicative structure.
  If $\mathbf{W}$ is a monoidal Waldhausen category then $\mathcal{D}_*\mathbf{W}$ is endowed with the structure of a quadratic pair algebra and hence by results of Baues-Jibladze-Pirashvili it represents the first Postnikov invariant of $K\mathbf{W}$ as a ring spectrum

$$k_1 = \{\mathcal{D}_*\mathbf{W}\} \in HML^3(K_0\mathbf{W}, K_1\mathbf{W}).$$

- Comments on the proof.
  For the proof we compute a small model of the fundamental 2-groupoid of $wS.\mathbf{W}$ by using an Eilenberg-Zilber-Cartier theorem for ∞-groupoids. Then we use Curtis's connectivity result to obtain nilpotency degree 2.

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The End

Thanks for your attention!

The trivial relations

- $[*] = 0$.
- $[A \xrightarrow{1_A} A] = 0$.
- $[A \xrightarrow{1_A} A \twoheadrightarrow *] = 0$, $[* \rightarrowtail A \xrightarrow{1_A} A] = 0$.

back

The boundary relations

- $\partial[A \xrightarrow{\sim} A'] = -[A'] + [A]$.
- $\partial[A \rightarrowtail B \twoheadrightarrow B/A] = -[B] + [B/A] + [A]$.

back

Composition of weak equivalences

- For any pair of composable weak equivalences $A \xrightarrow{\sim} A' \xrightarrow{\sim} A''$,

$$[A \xrightarrow{\sim} A''] = [A' \xrightarrow{\sim} A''] + [A \xrightarrow{\sim} A'].$$

back

- For any commutative diagram in $\mathbf{W}$ as follows

$$\begin{tikzcd} A \arrow[r, tail] \arrow[d, "\sim"] & B \arrow[r, two heads] \arrow[d, "\sim"] & B/A \arrow[d, "\sim"] \\ A' \arrow[r, tail] & B' \arrow[r, two heads] & B'/A' \end{tikzcd}$$

we have

$$\begin{aligned} &[A' \rightarrowtail B' \twoheadrightarrow B'/A'] \\ &[A \xrightarrow{\sim} A'] + [B/A \xrightarrow{\sim} B'/A'] \\ &+\langle [A], -[B'/A'] + [B/A] \rangle \quad = \quad [B \xrightarrow{\sim} B'] \\ &\qquad\qquad + [A \rightarrowtail B \twoheadrightarrow B/A]. \end{aligned}$$

back

Composition of cofiber sequences

- For any commutative diagram consisting of four obvious cofiber sequences in **W** as follows

$$\begin{tikzcd}
 & & C/B \\
 & B/A \arrow[r, tail] & C/A \arrow[u, two heads] \\
A \arrow[r, tail] & B \arrow[r, tail] \arrow[u, two heads] & C \arrow[u, two heads]
\end{tikzcd}$$

we have

$$\begin{aligned}
&[B \rightarrowtail C \twoheadrightarrow C/B] \\
&+[A \rightarrowtail B \twoheadrightarrow B/A] \quad = \quad [A \rightarrowtail C \twoheadrightarrow C/A] \\
&\qquad +[B/A \rightarrowtail C/A \twoheadrightarrow C/B] \\
&\qquad +\langle [A], -[C/A] + [C/B] + [B/A] \rangle.
\end{aligned}$$

back

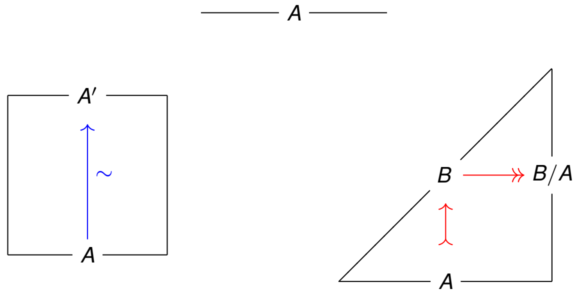
Coproducts

- For any pair of objects A, B in $\mathbf{W}$

$$\langle [A],[B]\rangle = -[A \overset{i_1}{\rightarrowtail} A \vee B \overset{p_2}{\twoheadrightarrow} B] + [B \overset{i_2}{\rightarrowtail} A \vee B \overset{p_1}{\twoheadrightarrow} A].$$

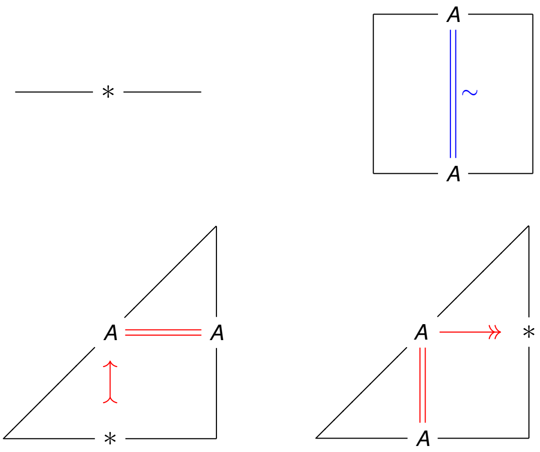
back

Bisimplices of total degree 1 and 2 in $wS.\mathbf{W}$

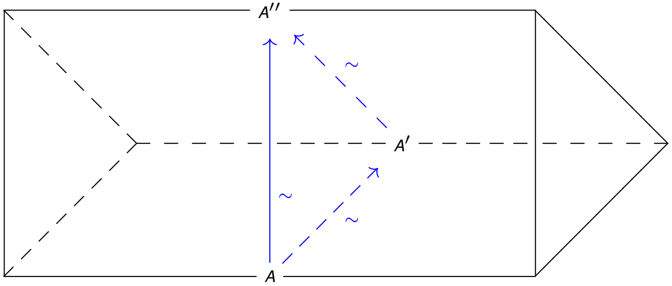


◂ back to generators ◂ back to relations

Degenerate bisimplices of total degree 1 and 2 in $wS.\mathbf{W}$



Bisimplex of bidegree $(1, 2)$ in $wS.\mathbf{W}$



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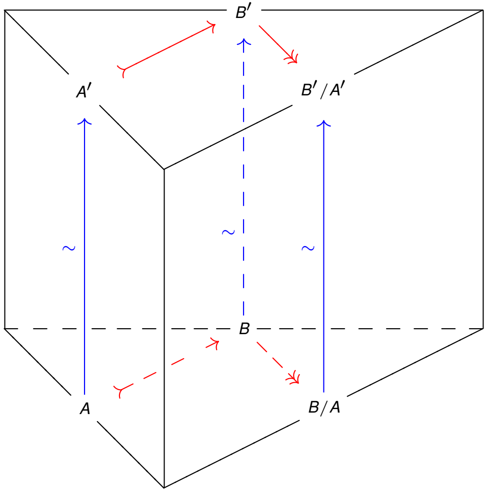
Bisimplex of bidegree (2, 1) in $wS.\mathbf{W}$



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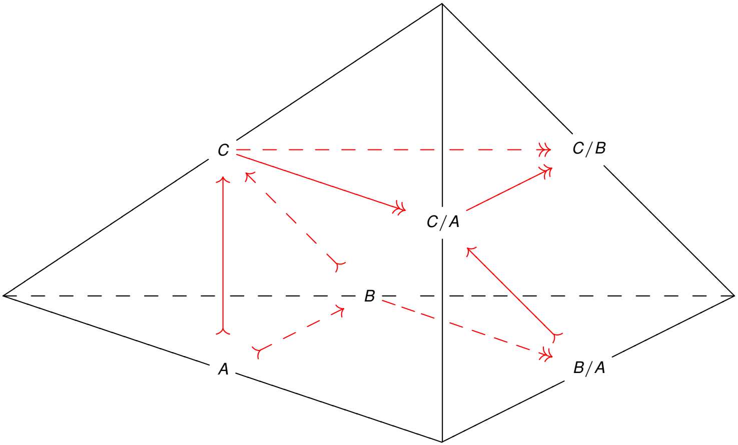
Bisimplex of bidegree $(3, 0)$ in $wS.\mathbf{W}$



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