Emergence of cooperation in an evolutionary game with two-level decisions

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Abstract

We introduce an extended version of the classical minority game model, with two groups of agents: a group of leading players and a group of following players. The members of the first group can be related to the financial "gooroos", who define market trends and are imitated by the members of the second group. This extension implements a two-level decision process, modeling a typical leadership behaviour of financial markets. We show by means of numerical experiments that the dynamics of this model leads to the emergence of coordination and organization patterns.

Key words: Minority game, Two-level decision

1 Introduction

In recent years, discrete adaptive games have attracted much attention because of their relation to economic and finantial markets. This kind of games generally involves a set of decision makers who, at discrete moments in time, independently choose an action from a finite set of available actions according to decision rules called strategies. The outcome of the game and the payoff that each agent receives depends on these individually chosen actions.

A well-known model dealing with this kind of behaviour is the *Minority Game* (MG), which is based on the *"El Farol"* bar problem [1] and was introduced in [2]. In that model, N(odd) agents must independently choose between two actions (usually denoted by 0 and 1), and the agents who made the minority decision win. Each agent's choice depends on a set of s strategies. Each strategy predicts the next winning action (0 or 1) by processing the outcomes

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from the last m time steps, which is the only available public information. The value m is known as the agents' *memory*. Since every strategy contains the 2^m possible historic states, the whole pool has 2^{2^m} strategies; and at the beginning of the game each agent randomly draws her set s from those (maybe repeated by chance). Eventually, those strategies that predict the correct outcome for each step are rewarded by one point, regardless of usage. Far away from the possibility of knowing her maximization-of-payoff choice, each agent is confined to play in the way her own best performing strategy (the one from her set with the best score up to that moment) suggests.
The MG is associated to the analysis of simple financial markets due to the similar kind of binary decisions that have to be made (e.g., "buy" or "sell") and to the bounded rationality and incomplete information that the individuals have. However, some other features may be taken into account in order to deal with more complex instances. In general, it is well known in economics that agents play in different ways: herd behavior, leadership, coordination, cooperation and competition have been widely recognized and modelled for real economy and financial markets [3–7].
In speculative businesses, at least two characteristic groups may take part as it was previously proposed in [8]: a *group of leading players* and a *group of following players*. The members of the first group can be related to the *gooroos*, who define financial trends and are closely followed by the members of the latter group, who consider that gooroos have better information than themselves.
In this paper we study a model based on the original MG that includes the above observations, by introducing agents acting as gooroos and others trying to emulate their actions. These rules establish a two-level decision scenario where one of the groups could take advantage of that. The proposed model is compared with other ways to play (introduced below), paying attention to the performance of each group and to the performance of the whole population as well as to the emergence of cooperation.

2 The Model

In this section we extend the MG with two kinds of decision makers. We assume that one of them holds similar characteristics to the original MG instance introduced in [2] and explained on the previous section (i.e., each player has s binary-prediction strategies and, for each time step, the strategies predicting the correct outcome gain one point). In contrast, the other group has non-binary strategies that refer to players belonging to the former group, and those predicting which agent from the first group will guess the next outcome gain one point. From now on, we call *leaders* the agents from the first group

Leader's Strategy		**Follower's Strategy**	
$m = 3$	Prediction	$m = 3$	Prediction
000	1	000	leader 02
001	1	001	leader 34
010	0	010	leader 18
011	1	011	leader 55
100	0	100	leader 71
101	0	101	leader 12
110	0	110	leader 09
111	1	111	leader 43

Table 1
Examples of typical strategies for a leader and a follower.

and *followers* the agents from the second group. Table 1 illustrates examples of two typical strategies for both leaders and followers. Under this rules, some configurations are forbidden, for instance, the extreme case with only one agent in the minority is only possible if the winner belongs to the leaders' group since any follower's decision is tied to a leader's one.

We consider agents playing with the same memory length m, the same value of choice (the choice of each agent weights one) and the same number s of strategies (drawn from their respective pools according to their groups) in order to assume similar capability of reasoning and focussing the difference between groups in the level of decision they make. Let us notice that the agents playing as followers have to deal with more complex rules than those of the leaders (since the prediction column is not binary), unless $L \leq 2$. The total number of available strategies is 2^{2^m} for leaders, L^{2^m} for followers and there are $N = L + F$ players interacting altogether, where L and F denote the number of leaders and followers, respectively.

Taking into account the payoff of the game, the optimal distribution of resources, from the society point of view, has $(N-1)/2$ winners without regard to the numbers of leaders and followers participating of the minority. Therefore, the smaller the number of winners, the bigger the social waste of resources due to the inefficient distribution of resources.

In the original MG, as well as in financial markets, the variance σ^2 of attendance of one of the two options is considered as one of the most important characteristics of the game because it measures the way that the population is globally wasting its resources. The smaller σ^2 is, the smaller the waste, the *risk* and the *volatility* of the market are. The emergence of coordination between agents in the MG is given by the fact that the agents collectively behave in a better way than randomly, in which case the variance is ${\sigma_r}^2 = N/4$. Indeed, in this case the coordinated agents' variance is $\sigma^2 < {\sigma_r}^2$ [9–12]. However, when

some players follow the others' choices, the game is not supposed to behave in the known classical way. It includes new critical variables to deal with and makes it necessary to redefine the concept of "random instance" to measure the coordination parameters.
We define the random instance as follows: in every time step each follower randomly chooses a leader to track, and then, each leader, trailing her set of followers, randomly chooses one of the two alternatives. It can be easily checked that in this case the variance is

$$\sigma_r{}^2 = \frac{L}{4}\left\{\left(\frac{F}{L}+1\right)^2 + \frac{F}{L}\left(1-\frac{1}{L}\right)\right\} \tag{1}$$

Notice that if only L agents with F weights (randomly distributed among them) played in the MG, the contribution of the average weight to $\sigma_r{}^2$ would be $(\frac{F}{L}+1)^2$ and the contribution of the distribution variance of weights would be $\frac{F}{L}(1-1/L)$.

3 Numerical Results

The aim of this section is to show the performance of the game for three kinds of followers:

i) *strategical followers* (SF): with followers playing according to the above introduced model;
ii) *fixed followers* (FF): with each follower randomly choosing at the beginning a leader to track along the whole game;
iii) *random followers* (RF): with each follower randomly choosing for each time step a new leader to track.

As the variance measures how well the agents are able to distribute resources, a key point is to observe where $\sigma^2 < \sigma_r{}^2$ holds under different conditions. For large L we have $(1-1/L) \sim 1$. Then, from Eq. (1) we get

$$\sigma_r{}^2 \sim \frac{L}{4}\left\{(k+1)^2 + k\right\} \tag{2}$$

where $k = F/L$. For the original MG (recoverable by the particular case $k = 0$ or equivalently $L = N$), the scaling factors $2^m/N$ and σ^2/N have been considered, for which the results for different values of m and N show the same curve [9,11,13]. In our model, a comparable scaling effect is obtained by using L instead of N, as inferred from Eq. (2). Since $\sigma_r{}^2/L$ only depends on k, we

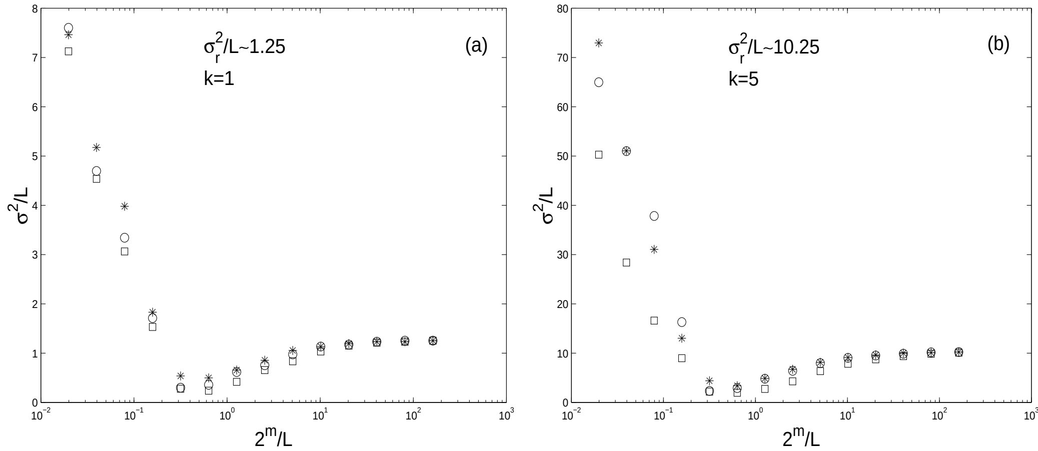

Fig. 1. σ^2/L as a function of $2^m/L$ for SF (squares), FF (circles) and RF (stars) behaviours with (a) $k = 1$ and (b) $k = 5$.

can study instances where the agents' behaviour evolves to achieve coordination for different values of k. We compare the variance σ^2/L of attendance to the alternative 0 as a function of $2^m/L$, with $s = 2$, for the three kinds of followers introduced above. Figs. 1(a) and (b) show the results for $k = 1$ and $k = 5$, respectively. It is observed that, as the information available to the agents increases (as $2^m/L$ grows), σ^2/L tends to the random instance value (Eq. (2)) due to their difficulty of managing that increasing amount of information [2]. We see that the best performance of agents (minimization of σ^2/L) is achieved when the followers play strategically (SF). This means that they help to improve the coordination when they also get an inductive learning. In this sense, the random movement of followers (RF) makes impossible their learning process and the game develops its worst performance. In spite of the three cases get a coordination that overcomes the random performance, the SF behaviour shows the highest levels of coordination. Therefore, we compare it to the original MG by means of an appropriate scaling according to the value of k. The aim is to compare the performance of the game for any k with an instance of a MG defined applying a fixed weigth $k+1$ to each agent. In this weighted MG (WMG) the "noise" generated by the followers is minimized and hence it is useful as a reference to compare with. The variance σ_r^2 of the random instance of the WMG is obviously given by $\frac{L}{4}(k+1)^2$, providing a scaling factor $1/\lambda = (k+1)^2$. It is shown in Fig. 2(a) the way the whole population develops an efficient coordination, for several values of k, when both leaders and followers learn. This coordination appears to be (around the minimum σ^2/L) even better than the WMG instance, but getting worse than that as $2^m/L$ grows, tending finally to $\lambda\sigma_r^2/L \sim \frac{1}{4}(1 + \frac{k}{(k+1)^2})$. Since this asymptotic value is grater than $1/4$ (for $k > 0$) a similar variance to that of the WMG, in the maximum coordination region, implies an interesting performance of the game.

This global behaviour does not bring any information about the benefits of

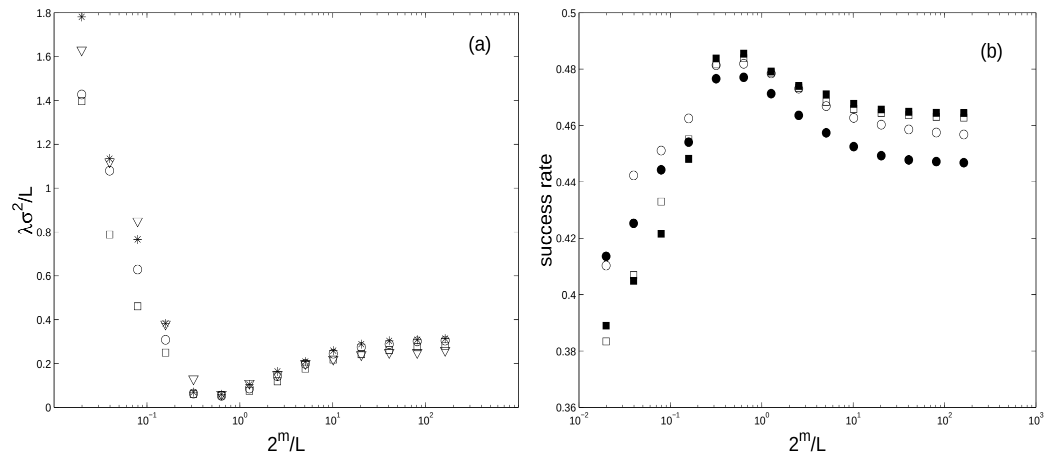

Fig. 2. (a) Rescaling of SF behaviour for λ corresponding to $k = 0$ (triangles), $k = 1$ (stars), $k = 2$ (circles) and $k = 5$ (squares). Notice that $k = 0$ is equivalent to the original MG (Eq. (2)). (b) Sucess rate for leaders (squares) and followers (circles) with SF behaviour, for $k = 1$ (filled) and $k = 5$ (empty). Notice the swap of overcoming group when the coordination starts.

each group. Regarding this point, the success rates for both leaders and followers are analyzed separatedly. Fig. 2(b) shows the leaders taking an advantage over the followers with SF behaviour, for different values of k, mainly due to the level of decisions that each group takes. However, this advantage decays as k grows, showing the difficulty for the leaders to manage the followers opinion. In fact, this tendency can be seen for different followers' behaviours as it is shown in Figs. 3 and 4 by the success rates of leaders and followers, respectively, for $k = 1$ and $k = 5$. Another important feature of Fig. 2(b) appears for small values of $2^m/L$, where $\sigma^2 > {\sigma_r}^2$. In that region of the figure, the followers' success rate overcomes the leaders' one, implying a phase transition in $\sigma^2 \sim {\sigma_r}^2$ ($2^m/L \sim 0.2$), regardless of k.

According to Fig. 3, the leaders always improve their success rate comparing when playing alone but, as mentioned above, that improvement decreases as k grows, stabilizing asymptotically in the original MG performance. Even when the global coordination behaves similarly to the origial MG, there is an unequal distribution that benefits to the leaders. In this sense, we see in Fig. 2(b) that the followers with SF behaviour approach the leaders' success rate only in the region of maximum coordination. In general, the followers' three behaviors differ significantly from the leaders' behavior, being more similar only when k is increased. It is clearly seen that the RF behaviour is the worst way to play because it not only generates the lowest followers' incomes, but also does not generate an observable increase of leaders' incomes. It should be noted that, in spite of the difference of success rate of followers' three behaviours, there are not big differences in the leaders' success rate. This suggests that, in the region of coordination, the leaders do not take extra profits based on the followers' incapability of learning, but only on the decision level and on the value of k. As mentioned above, where $\sigma^2 < \sigma_r^2$, the leaders' success rate

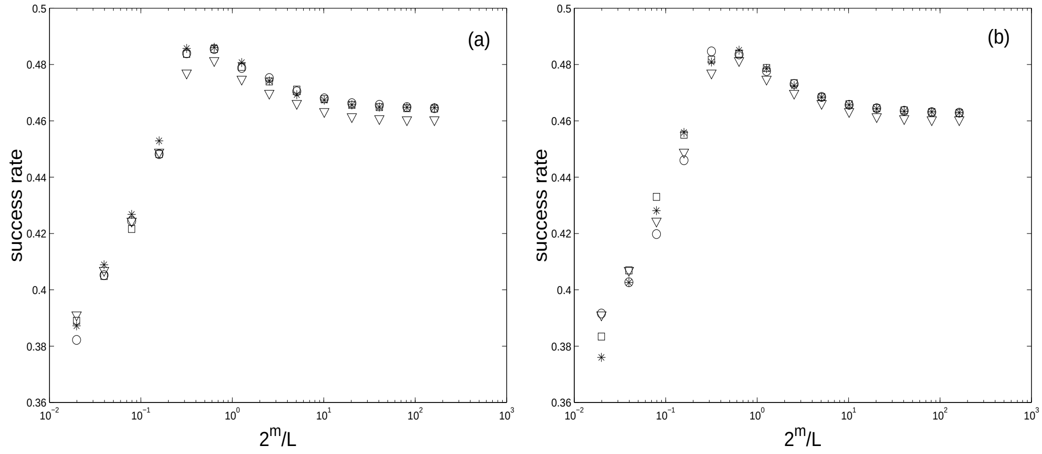

Fig. 3. Success rate of leaders for SF (squares), FF (circles) and RF (stars) behaviours with the original MG (triangles) as a comparison parameter for (a) $k = 1$ and (b) $k = 5$.

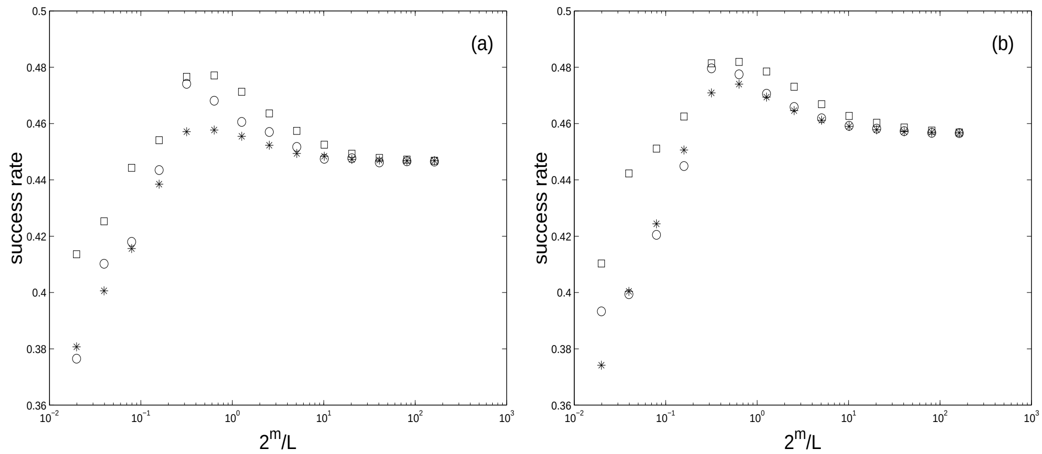

Fig. 4. Success rate of followers for SF (squares), FF (circles) and RF (stars) behaviours for (a) $k = 1$ and (b) $k = 5$.

tends to the original MG instance as k grows, that is, tends to the success rate of the $k = 0$ instance. Therefore, there should exist some $0 < k < \infty$ where the leaders' performance is optimal. Additional simulations showed that the optimal point corresponds to $k = 1$, regardless of $2^m/L$.

4 Conclusions

We have analyzed a generalization of the MG problem that introduces three central, heterogeneous entities: agents, decision levels and agents' way of deciding. These features distinguish this model from less realistic models about this topic and open the possibility of new interesting alternatives.
According to our results, in the better-than-random region, the leaders are

able to get better performance than the followers with similar intelligence conditions and than themselves when playing alone due to the different levels of decision between groups. Besides, SF behaviour is more successful than both FF and RF and shortens the distance between leaders and followers. However, this distance is also shortened by increasing k, regardless of the followers' behaviour. This means that, when k is small (optimized in $k = 1$) and the game is developed in a given range of public information (about $0.2 < 2^m/L$), leaders may read better the game and get benefits of it, which reveals the importance of how individuals manage their own information and the public one with respect to their decision level.
As a remarkable result, an efficient coordination is observed. On the one hand, this means that there is a "multi-level thinking" intra and inter groups with which both groups inductively learn to play. On the other hand, this coordination suggets a deep organization and an effective learning.
Finally, further research in this direction should be addressed to explain the reason of the observable phase transition when coordination starts. Other issue worth studying is how the characteristics of an evolutionary game with groups exchanging agents or with just one group of agents with mixed strategies would behave.

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