EXCEPTIONAL PLANAR WEBS AND THEIR ASSOCIATED (EXCEPTIONAL) SURFACES

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Codimension one d**-webs** are configurations of d codimension one foliations in general position. Much of the classical theory evolved around the concept of **abelian relation**: a functional relation between the first integrals of the foliations defining the web reminiscent of Abel's addition theorem in classical algebraic geometry. The abelian relations of a given d-web form a finite dimensional vector space whose dimension (the **rank** of the web) is bounded by Castelnuovo number $\pi(n, d)$ where n designate the dimension of the ambient space. A fundamental problem in web geometry is the classification of **exceptional webs**, that is webs of maximal rank that are not equivalent to a so-called **algebraic web** (a web associated via projective duality to an algebraic projective curve).

In this talk, we will focus on dimension 2, the simplest but richest situation since numerous planar exceptional webs have been recently discovered. We will present

- the construction of families of exceptional planar d-webs for every $d \geq 5$ (joint work with D. Marìn and J.V. Pereira);
- the classification of exceptional Completely Decomposable Quasi-Linear (CDQL) webs on $\mathbb{P}^2(\mathbb{C})$ and on 2-dimensional complex tori (joint work with J.V. Pereira).

Finally, if time allows, we will explain that, for exceptional webs, one can mimic the construction of the classical canonical map of an algebraic curve. Building on classical constructions of Italian geometers allows us to realize, in a canonical and geometrical way, exceptional webs on certain projective surfaces with very specific projective-differential geometry.