Mixed Lefschetz Theorems and Hodge Riemann Bilinear Relations: Geometry and Combinatorics

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Statements analogous to the Hard Lefschetz Theorem and the Hodge-Riemann bilinear relations hold in a variety of contexts: they impose severe restrictions on the cohomology algebra of a smooth compact Kaehler manifold or on the intersection cohomology of a projective variety; they restrict the local monodromy of a polarized variation of Hodge structure; they impose conditions on the possible f-vectors of convex polytopes. While the statements of these theorems depend on the choice of a Kaehler class, or its analog, there is usually a cone of possible Kaehler classes. It is then natural to ask whether these theorems remain true in a mixed context. In this talk I will explain the combinatorial motivation for such results and indicate how one can give a unified proof of them.